Sound Generation with Aperiodic Ordered Systems

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Abstract

Aperiodic 1D systems introduced in physics in the field of quasicrystals are used in this work, in order to generate selfsimilar aperiodic time structures. The Fourier analysis of series of impulses distributed in time in a non periodic but ordered way, shows that for some cases the spectrum has a discrete part that can be used for sound synthesis.

1 Introduction

The models used today for sound generation are basicaly of three types, namely instrumental, spectrum and abstract models. This work belong to the abstract models category.

In the last decade a new type of structures, lying somewhere between order and disorder, between periodicity and randomness, appeared in mathematical physics.

There are several methods to generate the structures: projection from a periodic lattice, use of substitution rules and others. In this work, we consider 1D aperiodic structures generated by substitution or production rules in a special type of formal grammars, known as D0L systems [1].

A very well known example of an aperiodic 1D structure is the Fibonacci sequence, with a distribution of points along the time axis according to [2]:

$$t_n=n+\alpha+[[n/\tau+\beta]]/\tau \qquad (1)$$

where [[x]] is the greatest integer less than x, $\tau=(1+\sqrt{5})/2\approx 1.618$ is the golden number, α and β are arbitrary real numbers, and n is a non-negative integer. This equation describes a sequence of points, such that the interval between two consecutive points can be only of two lengths: $l_1=\tau$ and $l_2=1$. They appear in a non periodic but ordered sequence, where the ratio of the number of l_1-intervals to the number of l_2-intervals is also equals to τ.

An equivalent way to define the Fibonacci sequence is through a D0L system which consists in an alphabet, a set of production rules for the allowed words, and an axiom or starting symbol. Consider the alphabet A,B, the production rules

$$A \to AB \qquad\qquad B \to A$$

and the axiom A. The language consists in the words A,AB,ABA,ABAAB,ABAABABA,....

If we associate to A and B two segments of lengths l_A and l_B with $l_A/ l_B =\tau$ we get the Fibonacci chain. The sequence is selfsimilar: a change of scale by a factor τ produces another sequence represented by the same word.

The Fibonacci chain can be used to structurate the time in a non periodic way [3]. Although the golden number is not rational, we can take rational approximants for it. If we define the Fibonacci numbers F_n as

$$F_{n+1}= F_n + F_{n-1}$$

with $F_0= F_1 =1$ it is very well known that

$$F_n/F_{n-1}\to\tau \text{ when } n\to\infty.$$

The sequence 2/1,3/2,5/3,8/5... can be used to generate aperiodic rhythms, with two rhythmic units A and B in a rational ratio.

Another example of a selfsimilar structure with the same basic cells can be obtained by applying the production rules

$$A \to AABA \qquad\qquad B \to BAB$$

The scaling factor in this case is $\tau+2$.
If we use the alphabet L,S,T, axiom L, and production rules

$$L\to LST,\ S\to LS,\ T\to L$$

the scale factor is $\theta=1+2\cos 2\pi/7 \approx 2.247$ if we take $l_L/l_S=2\cos 2\pi/7$ and $l_T/l_S=1-2\cos 3\pi/7$. We can get rational approximants by defining the numbers

$$L_{n+1}=L_n+S_n+T_n,\ S_{n+1}=L_n+S_n,\ T_{n+1}=L_n$$

with $L_0=1$, $S_0=0$, $T_0=0$ or also $L_0=0$, $S_0=1$, $T_0=0$.

The temporal structure derived from the substitution rules

$$L\rightarrow LSL,\ S\rightarrow LST,\ T\rightarrow ST$$

is also selfsimilar with scaling factor $\phi=1+2\cos\pi/7$. Observe that the word selfsimilarity is not being used in the same sens as for fractals. In fact there is always a minimum separation between points; there is not the ever finer detail that occurs in fractals. Obviously, infinite sequences of regularly spaced impulses are also selfsimilar, in the sens that if we replace each impulse by two impulses separated half the distance, the result is again an infinite sequence of regularly spaced impulses. But one of the most interesting properties of rhythms constructed with the aperiodic sequences considered in this work, is that they are not predictable: given a word it is not always possible to know what is the letter after a given one, without reproducing the whole word. Other substitutional sequences have been already used to produce musical shapes (see for instance [4]).

2 Time structures with discrete spectrum

A distribution of impulses on the points t_k of the time axis, with k integer, can be represented by the function

$$\rho(t)=\sum_k \delta(t-t_k) \qquad (2)$$

where the Dirac delta-function $\delta(x)$ has the properties: $\delta(x)=0$ unless $x=0$, $\delta(0)=\infty$.
Its Fourier transform is

$$\lim_{N\rightarrow\infty}((1/N)\sum_k \exp(it_k)) \qquad (3)$$

where N is the number of impulses. The Fourier transform of a sequence of impulses distributed along the time axis, is called the spectrum of the sequence, and is a sum of discrete and continuous components. The discrete component indicates order, the continuous component disorder. In this section we consider the aperiodic systems defined in section 1. The Fibonacci system and the LST-system with scaling factor θ, have discrete spectrum and therefore "enough order".

A periodic distribution with period of length a can be represented by the function

$$M_a(t)=\sum_k \delta(t-ka) \qquad (4)$$

If t denotes the time in seconds, then the Fourier transform of $M_a(t)$ is proportional to $M_{2\pi/a}(\omega)$ where ω denotes the frequency in Hertz. For instance if we take $a=\pi/33$ we get the harmonic series: $C_2,C_3,G_3,C_4,E_4,G_4,Bb_4,C_5,D_5,E_5$... all of them with the same amplitude, which is measured by the proportionality factor of the function $M_{2\pi/a}(\omega)$.
In the Fibonacci case the impulse distribution is

$$M_a(t)=\sum_n \delta(t-t_n) \qquad (5)$$

where t_n is given by equation (1). The spectrum of the distribution is discrete, and can be computed with the help of the golden number τ and two integers p and q ,through the following expression:

$$\omega_{pq}=(2\pi/(1+1/\tau^2))[p+q/\tau] \qquad (6)$$

and with a different amplitude for each component. If we take $X=2\pi q-\omega_{pq}/\tau$,then the amplitude is proportional to $(2\sin(X/2))/X$ (see *Figure1*).

A pitch defined by p and q is more intense if $\tau q-p$ is small or p/q close to τ, that is when p,q are successive Fibonacci integers F_n,F_{n-1}. Outside this sequence, the amplitudes decrease strongly and, above a certain amplitude threshold, the number of partials below a fixed frequency is always finite.

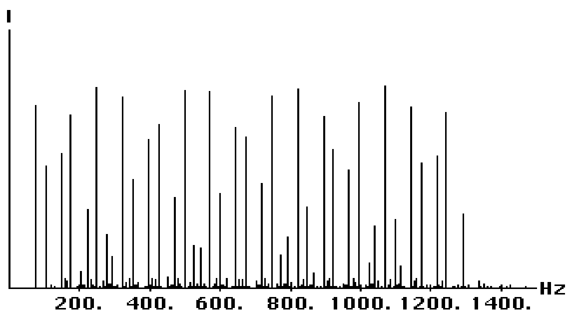

Figure 1. Spectrum for the Fibonacci system.

Given an arbitrary aperiodic system, it is not easy to find closed expressions for the amplitudes, as in the Fibonacci case. However recursion relations can be obtained allowing to get the Fourier amplitudes in an efficient way ([1] and references therein).

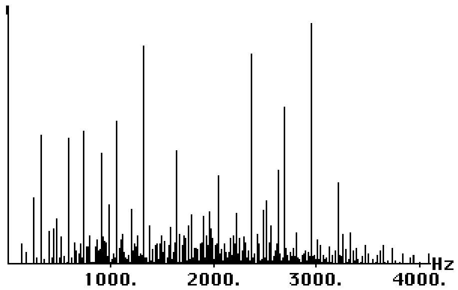

Figure 2. Discrete part of the spectrum for the L-S-T system with scaling factor θ

The order that lies behind the chain of temporal intervals, is reflected in the frequency distribution. In the AB-case it can be shown, that a set of partials separated by intervals of two lengths can be taken. The computations show that the ratio of the two intervals is the golden number (see the wavetable f1 in section.3, and compare the differences between consecutive partials). We can also extract from the spectrum of the first LST-system, a set of partials separated by intervals of three lengths, with the same ratio as l_L, l_S, l_T (see f2 in section.3).

The words obtained in the frequency space do not belong to the language generated by the grammar defining the temporal chains. In the Fibonacci words two B intervals can never be adjacent, nor can three A intervals. It can be seen that in the frequency space there are two consecutive short intervals.

3 Some examples

Due to the lack of translational symmetry, the spectra are always inharmonic. If the Csound language is used, the pairs frequency-amplitude for the Fibonacci system give the following composite waveform of weighted sums of simple sinusoids:

f1 0 4096 9 1 6.92784 0
2.618 19.4934 0 3.618 12.9569 0 5.236
14.398 0 6.236 18.5066 0 7.854 8.27371
0 8.854 21.363 0
9.854 5.67165 0 11.472 20.3011 0 12.472
11.49 0 14.09 15.7818 0 15.09 17.3651 0
16.7 9.6877 0 17.71 21.022 0 19.326
4.27182 0 20.326 20.9093 0 21.326
10.0286 0

where we remind that the three p-fields in GEN09 correspond to the partial, strength of the partial and initial phase. A function table for the first LST system is:

f2 0 4096 9 1 0.659804 0
2.24734 0.875313 0 3.24751 1.08665 0
4.49468 0.292446 0 5.04974 5.47652 0
6.29692 1.37356 0 7.29708 1.94143 0
8.29725 0.46514 0 9.54442 0.28039 0
10.0995 0.918011 0 11.3467 5.61295 0
12.3468 1.19516 0 13.594 1.06184 0
14.149 4.70316 0 15.3962 2.48997 0
16.3964 6.05437 0 17.3966 1.28956 0

Bell-like timbres can be generated with the classical additive synthesis technique due to J.C.Risset [5]. The amplitude peaks of the partials are taken from f1 and f2 with exponential decay, durations inverse to their frequencies and beatings of the lowest two partials.

Other examples can be obtained from additive synthesis of filtered noises with center frequencies located at multiples of the partials given in f1 and f2, appropriate bandwidths, and gaussian envelopes with peaks placed at different times according with other parameters like frequency or amplitude.

If the recursion relations are used, temporal evolutions of the different partials can be implemented also, by increasing the number of impulses in the time axis: the ratio of amplitudes to number of time points stabilizes, when we increase the number of iterations.

4 Conclusion

In this work, aperiodic ordered temporal structures having discrete inharmonic spectra, have been discussed. A criterion based in number theory has been given in [6] to characterize a system with discrete spectrum. According with their results, the AB-system with scaling factor $\tau+2$ and the LST-system with scaling factor ϕ have no discrete part in their Fourier transforms.

In the compositional level, selfsimilarity suggest that musical form can also be articulated with the help of these multilevel hierarchies. The underlying grammar structure can represent connections between different sections of music having a high structural generality. Also the range of raw musical data that can be represented is high. The rhythmic structure shows both autonomy and solidarity with the pitch-intensity material.

Although only 1D examples have been considered in this work, the analysis of aperiodic structures in 2D and 3D [1] are also a rich source of musical applications. Some of them have discrete Fourier components and can be described in terms of formal grammars.

Spectral modeling of musical sounds (SMS) representing sinusoids and noise as two separate components has been developed in the last years (see [7] and references therein). The analysis detects time varying sinusoids which corresponds ,in our terminology, to the discrete part of the spectrum. These partials are then subtracted to obtain the noise component. This technique can be used also in our model in order to get the (static) continuous part of the spectrum. The use of aperiodic systems in combination with SMS and other techniques, like wave terrains or granular synthesis [8], could give results of interest.

The ideas presented in this work have been the basis of *Estudio del Tiempo Iluminado II* for tape, realized in the authors personal studio and in the LIEM-CDMC (Centro de Arte Reina Sofía,Madrid 1997).

References

[1] García Escudero, J. 1997. "Formal Languages for Quasicrystals", in *Symmetries in Science IX*, Plenum Press. New York, p.139.

[2] Levine, D. and Steinhardt, P. 1986. "Quasicrystals. I. Definition and structure." *Physical Review,* B.34, p.596.

[3] García Escudero, J. 1995. "Images from the Aperiodic Time." in *Anais II Simpósio Brasileiro de Computaçao e Música*. Instituto de Informatica da Universidade Federal RGS.Porto Alegre.p.62

[4] Allouche, J. P. and Johnson, T. 1995. "Finite Automata and Morphisms in Assisted Musical Composition". *Journal of New Music Research*, p.97

[5] Risset, J.C. 1969 *Introductory Catalogue of Computer-Synthesized Sounds*. Murray Hill, NJ. Bell Telephone Laboratories.

[6] Bombieri, E. and Taylor,J.E. ,1986, "Which Distributions of Matter diffract? An initial investigation", *Journal de Physique France*, C3, p.19

[7] Serra, X., 1997 "Musical Sound Modeling with Sinusoids plus Noise", in *Musical Signal Processing*. Swets&Zeitlinger Publ.

[8] Roads C. 1996. *The Computer Music Tutorial*. MIT Press.