Dynamic Models of Pseudo-Periodicity

Pietro Polotti
Gianpaolo Evangelista

Laboratoire de Communications Audiovisuelles (LCAV)
École Polytechnique Fédérale de Lausanne, Switzerland
pietro.polotti@epfl.ch *gianpaolo.evangelista@epfl.ch*

Abstract

Voiced musical sounds have non-zero energy in sidebands of the frequency partials. Our work is based on the assumption, often experimentally verified, that the energy distribution of the sidebands is shaped as powers of the inverse of the distance from the closest partial. The power spectrum of these pseudo-periodic processes is modeled by means of a superposition of modulated 1/f components, i.e., by a pseudo-periodic 1/f –like process. Due to the fundamental selfsimilar character of the wavelet transform, 1/f processes can be fruitfully analyzed and synthesized by means of wavelets, obtaining a set of very loosely correlated coefficients at each scale level that can be well approximated by white noise in the synthesis process.

Our computational scheme is based on an orthogonal P-band filter bank and a dyadic wavelet transform per channel. The P channels are tuned to the left and right sidebands of the harmonics so that sidebands are mutually independent. The structure computes the expansion coefficients of a new orthogonal and complete set of Harmonic Wavelets. The main point of our scheme is that we need only one parameter in order to model the stochastic fluctuation of sounds from a pure periodic behavior.

Keywords: wavelets, 1/f-noise, spectral modeling

1 Introduction

The purpose of this work is to introduce a technique for the analysis and synthesis of pseudo-periodic signals based on a special kind of wavelet packets, i.e., the Harmonic Wavelet Transform. We start from the wavelet-based model for 1/*f* processes introduced by Wornell [4] and we extend this model to the pseudo-periodic 1/*f*-like signals.

1/*f* processes can be employed for representing chaotic systems that are strongly influenced by their past behavior [1-3]. Voiced sounds in speech and music are pseudo-periodic signals and exhibit a long-term correlation or more precisely an approximate 1/*f* behavior in the neighborhood of each harmonic partial.

This is due to the chaotic but correlated micro-fluctuations from the periodic behavior of the signal itself. These fluctuations play a relevant role in the emulation of naturalness of voiced sounds.

Wornell in Theorem 3 in [4] states that a process

$$x(t) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x_n(m) \psi_{n,m}(t) \,,$$

where $\psi_{n,m}(t)$ form a orthonormal wavelet basis and the $x_n(m)$ are collections of mutually uncorrelated zero-mean coefficients, is nearly 1/*f*, i.e., its time-averaged power spectrum

$$\overline{S_x}(\omega) = \sigma^2 \sum_{n=-\infty}^{\infty} 2^{\gamma n} \left|\Psi\left(2^n \omega\right)\right|^2$$

satisfies the relations

$$\frac{\sigma_{L,q}{}^2}{|\omega|^\gamma} \le \overline{S_x}(\omega) \le \frac{\sigma_{U,q}{}^2}{|\omega|^\gamma} \tag{1}$$

for some $0 < \sigma_L^2 \le \sigma_U^2 < \infty$

We will extend this result to pseudo-periodic signals, introducing the Harmonic Wavelets, which consists of a continuos-time infinite-channel filter bank and a discrete wavelet transformation of each channel.

The most important result of this synthesis technique is that it allows one to control a highly complex stochastic process by means of relatively few parameters.

In section 2 we define the pseudo-periodic 1/*f* noise by means of a Harmonic Modulation and Demodulation scheme. In section 3 we illustrate the theoretical results on which our new method of synthesis is based. In section 4 we briefly review the Discrete Harmonic Wavelets and their properties. Section 5 deals with some applications to music synthesis.

2 Pseudoperiodic 1/f –like noise

We now provide a formal definition of pseudoperiodic 1/*f*-like noise based on a general modulation and demodulation scheme.

The frequency spectra of pseudoperiodic signals are characterized by harmonically spaced peak at frequencies $\overline{\omega}_k = \frac{2\pi k}{T_P}$, where T_P is the average period of the signal. In order to separate the contribution of each of the harmonic bands one can devise a set of ideal narrow-band filters of bandwidth $\Delta\omega = \frac{\pi}{T_P}$ each fitting a single sideband of the harmonics. The magnitude frequency response of these filters is given by

$$H_q(\omega)=\begin{cases}\chi_{\left[\frac{q\pi}{T_P},\frac{(q+1)\pi}{T_P}\right[}(\omega) & q\ge 0\\ \chi_{\left]\frac{q\pi}{T_P},\frac{(q+1)\pi}{T_P}\right]}(\omega) & q<0\end{cases},\quad q=0,\pm1,\pm2,\ldots,$$

where

$$\chi_{[A,B[}(\omega)=\begin{cases}1 & \text{if } A\le\omega<B\\ 0 & \text{otherwise}\end{cases}$$

is the characteristic function of the interval $[A,B[$. The outputs of these filters may be baseband shifted, according to a suitable demodulation scheme. This results in the demodulation scheme reported in Fig. 1.

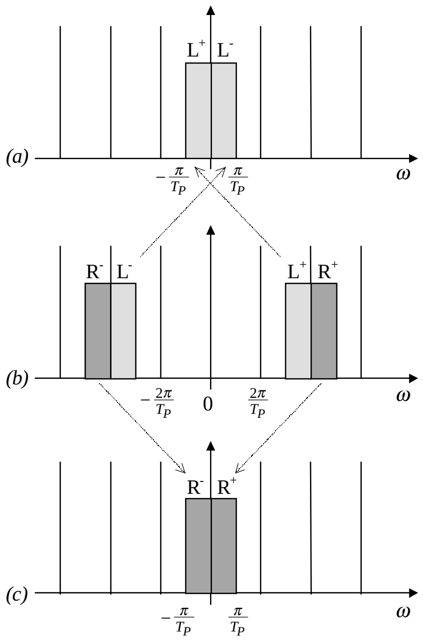

Fig. 1: *Baseband shift of harmonic sidebands: (b) sidebands of the 2nd harmonic; (a) demodulation of the left sidebands; (c) demodulation of the right sidebands.*

Demodulation of a signal $x(t)$ in L^2 may be described by the scalar products

$$w_q(t)=\int_{-\infty}^{\infty}K_q(t,\tau)x(\tau)d\tau\,,\quad q=0,1,\ldots,$$

where the kernels of the set of linear operators $\mathbf{K}_q$, q=0,1,... , have the form

$$K_q(t,\tau)=\frac{1}{T_P}\cos\left(\frac{t-(-1)^q(2q+1)\tau}{2T_P}\pi+\beta_q\right)\operatorname{sinc}\left(\frac{t-\tau}{2T_P}\right)\qquad(2)$$

where $\beta_q=\beta_{-q-1}$ are arbitrary phase factors.

The operators (2) have support in

$$W_q=\left]\frac{-(q+1)\pi}{T_p},\frac{-q\pi}{T_p}\right]\cup\left[\frac{q\pi}{T_p},\frac{(q+1)\pi}{T_p}\right[$$

and perform a harmonic demodulation to baseband $\left]\frac{-\pi}{T_P},\frac{\pi}{T_P}\right[$ of the signal subband W_q. The operator $\mathbf{K}_q$ is invertible, with inverse

$$K_q^{-1}(t,\tau)=K_q(\tau,t)=K_q^{\dagger}(t,\tau)\,,$$

where the symbol † denotes the adjoint. Thus $\mathbf{K}_q$ is unitary and the operators $\mathbf{K}_q^{-1}$ perform a harmonic modulation from baseband to V_q where V_q is the L^2 subspace of signals bandlimited to W_q.

We can thus model acoustic pseudoperiodic signals with fundamental frequency $f_0=\omega_0/2\pi$ by means of a superposition of harmonic modulated bandlimited $1/f$ processes. Each process has bandwidth equal to half the harmonic spacing ω_0 and contributes to a single side band of each of the harmonics.

Denoting by k the harmonics index and by L and R the left and right sideband, respectively, the average spectrum of a pseudoperiodic $1/f$ -like process has the following form for $\omega\ge 0$:

$$S(\omega)=\sum_{q=0}^{\infty}\frac{\sigma_{q,R}^2}{|\omega-q\omega_0|^{\gamma_{q,R}}}\chi_{[q\omega_0,(q+1/2)\omega_0[}(\omega)+\frac{\sigma_{q,L}^2}{|\omega-q\omega_0|^{\gamma_{q,L}}}\chi_{[(q-1/2)\omega_0,q\omega_0[}(\omega)\,,\qquad(3)$$

where $\sigma_{k,R}^2$ and $\sigma_{k,L}^2$ are the amplitudes and $\gamma_{k,R}$ and $\gamma_{k,L}$ the decay parameters.

Defining an ideal bandpass filter

$$H^{(\varepsilon)}(\omega)=\chi_{[-\omega_0/2,-\varepsilon]}(\omega)+\chi_{[\varepsilon,\omega_0/2]}(\omega)\,,\qquad(4)$$

where ε is arbitrarily small, we arrive at the following

Definition 1. A stochastic process $x(t)$ is said to be a $1/f$ -like pseudoperiodic noise if there exists a $T_P>0$ such that when $x(t)$ is operated by $\mathbf{K}_q$ in (2), yields a collection of processes

$$w_q(t)=\int_{-\infty}^{\infty}K_q(t,\tau)x(\tau)d\tau\,,\quad q=0,1,\ldots\qquad(5)$$

such that, when filtered through $H^{(\varepsilon)}(\omega)$, with $\omega_0=\frac{2\pi}{T_P}$, they become wide-sense stationary and bandlimited with power spectrum

$$S_{w_q}(\omega)=\begin{cases}\sigma_q^2/|\omega|^{\gamma_q} & \text{if}\quad \varepsilon<|\omega|<\omega_0/2\\ 0 & \text{otherwise}\end{cases},\qquad(6)$$

for some γ_q and σ_q.

The operations involved in (5) are equivalent to filtering the single sidebands of each of the harmonics, separately for the positive and negative frequencies, and properly baseband shifting the result.

Comparing (6) with the model spectrum in (3), we can make the following associations:

$\gamma_{2q-1}=\gamma_{q,L}$, $\gamma_{2q}=\gamma_{q,R}$, $\sigma_{2q-1}=\sigma_{q,L}$, $\sigma_{2q}=\sigma_{q,R}$.

Since the resulting processes $w_q(t)$ in Definition 1 are bandlimited to $[-\omega_0/2,\omega_0/2]$, they can be sampled with sampling rate $\frac{\omega_0}{2\pi}=\frac{1}{T_P}$.

For our purposes we introduce then the set of functions $\{g_{q,k}(t)\}_{q=0,1,\ldots;\,k\in\mathbf{Z}}$,

$$g_{q,k}(t) = g_{q,0}(t - kT_P) \,, \tag{7}$$

with

$$g_{q,0}(t) = \frac{1}{\sqrt{T_P}} \cos\left(\frac{2q+1}{2T_P}\pi t\right) \mathrm{sinc}\left(\frac{t}{2T_P}\right) ,$$

which is easily shown to form an orthonormal basis. It is easy to see that the scalar product (5) followed by sampling are equivalent up to a multiplicative constant $\sqrt{T_P}$ to the projection of $x(t)$ over the basis.

3 Synthesis of pseudoperiodic 1/f-like noise by means of Harmonic Wavelet Transform

In the Introduction we recalled Wornell's results about the synthesis of $1/f$ processes by means of Wavelet basis. We will provide now an equivalent result for the pseudoperiodic case. We need to prove a Lemma first.

In order to do this we define a new continuous- time Multiwavelet basis which we will call Harmonic Wavelet Transform (HWT):

$$\xi_{n,m,q}(t) = \sum_r \psi_{n,m}(r) g_{q,r}(t) \tag{8}$$

where the $\psi_{n,m}(r)$ form an ordinary Discrete Wavelet basis and the $g_{q,r}(t)$ are defined according to (4).

The Fourier transforms of Harmonic Wavelets are:

$$\Xi_{n,m,q}(\omega) = \Psi_{n,m}(P\omega) H_{q,0}(\omega) \,. \tag{9}$$

The action of filtering is essentially that of selecting a single sideband of the harmonics. Then we can prove the following:

Lemma 2

A signal x(t) such that

$$x(t) = \sum_{q=0}^{\infty}\sum_{k=-\infty}^{\infty} v_q(k) g_{q,k}(t) = \sum_{q=0}^{\infty}\sum_{n=1}^{N}\sum_{m=-\infty}^{\infty} \beta_q^{n/2} v_q^n(m) \xi_{n,m,q}(t) ,$$

where the $\xi_{n,m,q}(t)$ *are defined in the (8), the* $\{v_q(k)\}$ *are WSCS processes of the kind of:*

$$v_q(r) = \sum_{n=1}^{N}\sum_{m=-\infty}^{\infty} \beta_q^{n/2} v_q^n(m) \psi_{n,m}(r) , \tag{10}$$

the $v_q^n(m)$ *are mutually uncorrelated coefficients and* $\beta_q = \sigma_q^2 2^{\gamma_q}$, *is cyclostationary with period* $2^N T_P$. *The same result holds for the scale residue of the scale-truncated expansion.*

For the synthesis we derived the following

Proposition 3

The random-process

$$s(t) = \sum_{q=0}^{\infty}\sum_{m=-\infty}^{\infty} v_q(m) g_{q,m}(t) \;, \tag{11}$$

where the $\{g_{q,k}(t)\}_{q=0,1,\ldots;\,k\in\mathbf{Z}}$ *form an orthonormal set of functions, as defined in (7) and the* $v_q(m)$ *are, up to a multiplicative constant* $\sqrt{T_P}$, *the samples of approximately 1/f processes synthesized by means of a DWT filter bank, yields an average power spectrum of the form:*

$$\begin{aligned} \bar{S}(\omega) = \frac{1}{T_P} \sum_{q=0}^{\infty} \sigma_q^2 \left|G_{q,0}(\omega)\right|^2 \\ \times \left(\sum_{n=1}^{N} 2^{n\gamma_q} \left|\Psi_{n,0}(\omega T_P)\right|^2 + 2^{N\gamma_q} \left|\Phi_{n,0}(\omega T_P)\right|^2 \right) \end{aligned} \tag{12}$$

In the ideal case

$$\left|G_{q,0}(\omega)\right|^2 = \left(\chi_{\left]\frac{-(q+1)\pi}{T_P}, \frac{-q\pi}{T_P}\right]}(\omega) + \chi_{\left[\frac{q\pi}{T_P}, \frac{(q+1)\pi}{T_P}\right[}(\omega) \right)$$

and (12) is approximately 1/f near each harmonic $k\frac{2\pi}{P}$,

with $k = \left\lfloor \frac{q+1}{2} \right\rfloor$, *q=0,...,P-1 ; i.e., for*

$$(2k-1)\frac{\pi}{P} \le \omega \le 2k\frac{\pi}{P} \quad \textit{if q is odd,}$$

or for

$$2k\frac{\pi}{P} \le \omega \le (2k+1)\frac{\pi}{P} \quad \textit{if q is even,} \tag{13}$$

we have

$$\frac{\sigma_{L,q}^{\;2}}{\left|\omega - 2k\frac{\pi}{P}\right|^{\gamma_q}} \le \bar{S}_N(\omega) \le \frac{\sigma_{U,q}^{\;2}}{\left|\omega - 2k\frac{\pi}{P}\right|^{\gamma_q}}$$

for some $0 < \sigma_{L,q}^2 \le \sigma_{U,q}^2 < \infty$.

For the proofs of Lemma 2 and Proposition 3 see [8].

4 Discrete Harmonic Wavelets

The discrete counterpart of (7) is given by the basis associated with an ideal P band filter bank. We want to obtain an efficient scheme for the analysis and synthesis of pseudoperiodic $1/f$ noise. Thus we consider an approximation of the ideal filter bank granting perfect reconstruction. In particular, we consider the class of Type IV cosine modulated bases:

$$h_{q,r}(l) = h_{q,0}(l - rP) \;, \qquad q = 0,\ldots,P-1; \quad r \in \mathbf{Z}$$

$$h_{q,0}(l) = W(l)\cos\left(\frac{2q+1}{2P}\left(l - \tfrac{M-1}{2}\right)\pi - (-1)^q \tfrac{\pi}{4}\right) ,$$

where the lowpass prototype impulse response $W(l)$ of length M satisfies some technical conditions [8]. It is easy to prove that the set is orthogonal and complete.

In order to synthesize the samples of $1/f$ processes $w_q(k)$ we adopt the scheme devised by Wornell [4], which consists of an ordinary discrete wavelet synthesis structure, with white noise inputs. The overall structure is realized by introducing the Discrete Harmonic Wavelets [9]. These constitute a special type of multiwavelets, generalizing the Pitch-Synchronous Wavelet Transform class [6,7], defined by

$$\xi_{n,m,q}(k) = \sum_r \psi_{n,m}(r) h_{q,r}(k) \tag{15}$$

where $\psi_{n,m}(r)$ are discrete-time ordinary wavelets [5]. The Fourier transforms of the basis elements (15) are shown in Fig. 2.

In the analysis structure of the Discrete Harmonic Wavelet Transform, the signal is sent to a P channel filter bank and each output is Wavelet transformed (*WT* block). Signal reconstruction is achieved by separately inverse Wavelet transforming the Harmonic Wavelet coefficients and passing these sequences through the inverse P channel filter bank. This technique generalizes the one presented in [10] since independent control to each subband is allowed.

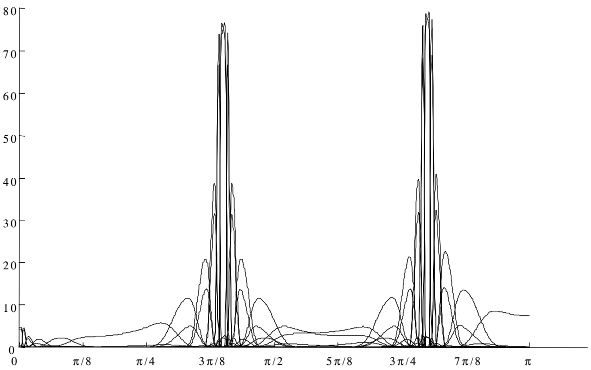

Fig. 2 : *Magnitude Fourier transform of Harmonic Wavelets (P=5).*

5 Applications to Music Synthesis

Our synthesis technique requires the estimation of three parameters per each harmonic partial k: σ_k , $\gamma_{k,R}$, $\gamma_{k,L}$. The parameters σ may be estimated from the frequency spectrum by means of a peak-picking algorithm. The estimation of the parameters γ_k is may be based on the HWT analysis. It is necessary to perform a linear regression according to the following law:

$$\log_2(Var(x_{n,m,q})) = \gamma_q n + \text{const} \quad (16)$$

with $k = \left\lfloor \frac{q+1}{2} \right\rfloor$.

Our experimental results show that HWT analysis supports the hypothesis that the shape of the side-bands of voiced sounds have an approximate $1/f$ behavior. In fact by linear regression we obtain correlation coefficients in the range 0.8 - 0.9 for the most relevant harmonics. Estimation improves if the bootstrap algorithm is employed to simulate a larger data set. We have then considered different samples of real instruments. The reproduction of the energy of the harmonic bands and of the sidebands in the synthetic sounds is well performed by our method, as shown in Figs. 3 and Fig. 4.

6 Conclusions

In this paper we introduced a new method for sound synthesis that allows us to control and reproduce the micro fluctuations present in real life voiced sounds. This method is a sort of additive synthesis where one adds modulated $1/f$ signals instead of pure sinusoidal functions. We defined a new class of stochastic processes, i.e., pseudoperiodic $1/f$-like noise. We introduced a special type of multiwavelet transform, i.e., the Harmonic Wavelet Transform. We devised an efficient analysis/synthesis scheme able to perform parameter estimation and generate pseudoperiodic $1/f$ -like noise.

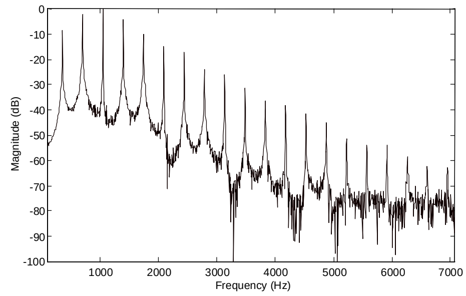

Fig. 3 *Frequency spectrum of real-life oboe (287.5 Hz)*

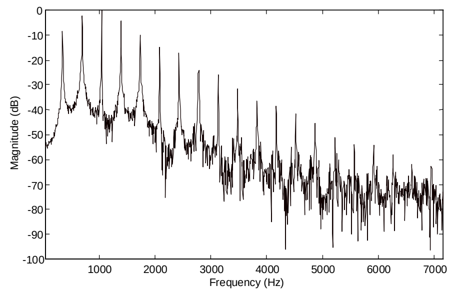

Fig. 4 *Frequency spectrum of synthesized oboe.*

References

[1] D. T. Gillespie, "The Mathematics of Brownian Motion and Johnson Noise," *Am. J. Phys.*, Vol. 64, pp. 225-240, March 1996.

[2] P. Flandrin "Wavelet Analysis and Synthesis of Fractional Brownian Motion," *IEEE Trans. on Information Theory*, Vol. 38, No 2, pp. 910-917, March 1992.

[3] M. Deriche and A.H. Tewfik, "Signal Modeling with Filtered Discrete Fractional Noise Processes," *IEEE Trans. on Signal Processing*, Vol. 41, No 9, Sept. 1993, pp. 2839-2849.

[4] G. W. Wornell, "Wavelet-Based Representations for the 1/f Family of Fractal Processes," *Proc. IEEE*, Vol. 81, No. 10, pp. 1428-1450, Oct. 1993.

[5] I. Daubechies, *Ten Lectures on Wavelets,* SIAM CBMS series, Apr. 1992.

[6] G. Evangelista, "Comb and Multiplexed Wavelet Transforms and Their Applications to Signal Processing," *IEEE Trans. on Signal Processing*, vol. 42, no. 2, pp. 292-303, Feb. 1994.

[7] G. Evangelista, "Pitch Synchronous Wavelet Representations of Speech and Music Signals," *IEEE Trans. on Signal Processing*, special issue on Wavelets and Signal Processing, vol. 41, no.12, pp. 3313-3330, Dec. 1993.

[8] P. Polotti, G. Evangelista, "Fractal Synthesis of Speech and Music by means of Harmonic Wavelets," technical report EPFL-LCAV.99ds

[9] G. Evangelista, P. Polotti, “Analysis and synthesis of pseudoperiodic 1/f-like noise by means of multiband wavelets,” *Proc. of XII CIM*, pp.35-38, Sept. 1998.
[10] G. Evangelista, “Pseudo-Periodic 1/f-Like Noise,” *Proc. of TFTS’96*, pp. 121-124, June 1996.