GENERALIZATION OF A 3D-RESONATOR MODEL FOR THE SIMULATION OF SPHERICAL RESONATORS

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ABSTRACT

A rectangular enclosure has such an even distribution of resonances that it can be accurately modeled using a feedback delay network, but a non rectangular shape such as a sphere has resonances that are distributed according to the extremal points of the spherical Bessel functions. This work proposes an extension of the already known feedback delay network structure to model a non rectangular shape such as a sphere. A specific frequency distribution of resonances can be approximated, up to a certain frequency, by inserting an allpass filter of moderate order after the delay line within the comb filter structure. The feedback delay network used for rectangular boxes is therefore augmented with a set of allpass filters allowing parametric control over the enclosure size and the boundary properties. This work was motivated by informal listening tests which have shown that it is possible to identify a basic shape just from the distribution of its audible resonances.

1. INTRODUCTION

The feedback delay network (FDN) of order N is a structure commonly used in artificial reverberation [1, 2, 4] to simulate the distribution of normal modes of vibration in an enclosure. In a rectangular enclosure, the distribution of normal modes can be obtained as the composition of (infinite) harmonic series, each series being associated with the spatial direction of propagation of the plane wave fronts supporting the modes. For instance, the longitudinal size of a rectangular box is associated with a low-pitch mode and with all its multiples. Since any harmonic series of resonances can be reproduced by means of a recursive comb filter, a reference FDN can be constructed as a parallel of comb filters or, in other words, with a diagonal feedback matrix. For the rectangular enclosure, the delay lengths can be computed exactly from the geometry of the room.

Non-rectangular enclosures usually do not have an even distribution of resonances. In some relevant cases, however, the modal distribution can be calculated in closed form from the geometric specification of the enclosure. In particular, this paper deals with the spherical resonator, whose resonances can be found by computing the local extremal points of Bessel functions. The spherical Bessel functions tend to cosine functions for larger values of the argument. A prior realization of the spherical resonator exploited the fact that the extremal points are asymptotically equidistant, using recursive comb filters with feedback high-pass filters to reproduce the medium- and high-frequency resonances [5]. On the other hand, each low-frequency resonance was reproduced by a tuned second-order resonant filter. Such prior realization was successfully experimented in the AML | Architecture and Music Laboratory, a museum installation where the visitor can experience how shapes such as, e.g. a tube, a cube or a sphere imprint a specific signature on the sounds. Informal reports from many listeners convinced us that it is indeed possible to identify basic shapes from the kind of resonance distribution they display.

Whereas the models used within the AML are specific to each shape, we try here, starting from the Ball-within-the-Box (BaBo) model [4], to design a single model valid for all shapes. The BaBo model was initially designed for rectangular shapes but we extend it to the simulation of non rectangular ones, in the hope that we can even feature a “shape control handle”. This paper reports the extension of the BaBo model to spherical enclosures and compares the audible results with recordings made through acoustical resonators.

2. RECTANGULAR RESONATOR MODEL

The BaBo model provides parametric control over the geometric and physical properties of a rectangular enclosure [4]. Kernel of the model is a feedback delay network where the delay lines have length in seconds given by

$$length = \frac{2}{c\sqrt{(l/X)^2 + (m/Y)^2 + (n/Z)^2}} \qquad (1)$$

where c is the speed of sound and l, m, n are triplets of small positive integers sharing no common (nontrivial) divisor.

If the feedback matrix is diagonal we have a parallel of comb filters and this corresponds to a perfectly reflecting enclosure. In this case the model is a parallel of comb filters, each comb representing the triples $(l,\ m,\ n)$, $(2l,\ 2m,\ 2n)$, $(3l,\ 3m,\ 3n)$, etc. thus giving a perfectly harmonic series of resonances. Whereas this structure is well suited for imitations of harmonic resonances, not all shapes can be characterized by harmonic series of resonances as we will see in the next section.

3. ACOUSTICS OF THE SPHERE

The modes f_{ns} of a sphere are proportional to the roots z_{ns} of

$$j'_n(x) = 0 \ , n = 0, 1, \ldots \tag{2}$$

where j_n is the spherical Bessel function[1] of order n and z_{ns} is the s^{th} root of the j'_n function. The theoretical resonance frequencies of the sphere are

$$f_{ns} = \frac{c}{2\pi a} z_{ns} \tag{3}$$

where c is the propagation speed of sound and a is the radius of the sphere [6].

Some roots z_{ns} of equation (2) are given in Table 1. S-ince the envelope of the sphere might be vibrating as well as dissipating some acoustic energy, these frequencies should be corrected for the effects that occur at the boundary [6, 3].

A closer look at the set of roots shows that they are not uniformly distributed, unlike the resonances in tubes or between parallel boundaries. The roots are wider apart at low frequencies than at high frequencies. This effect is stronger for higher values of n but any series of roots tend to be periodic in π for higher z values. This can be interpreted as a dispersion at low frequencies and will give us a hint how to implement the spherical resonator.

n \ s	1	2	3	4
0	0.00	4.49	7.73	10.90
1	2.08	5.94	9.21	12.40
2	3.34	7.29	10.61	13.85
3	4.51	8.58	11.97	15.24
4	5.65	9.84	13.30	16.61
5	6.76	11.07	14.59	17.95
6	7.85	12.28	15.86	19.26
7	8.93	13.47	17.12	20.56
8	10.01	14.65	18.36	21.84
9	11.08	15.82	19.58	23.11

Table 1: *Roots of* $j'_n(x) = 0$ *for order* $n = 0, \ldots, 9$ *and root number* $s = 1, \ldots, 4$.

4. MEASUREMENTS

Data are available from 3 experiments: Moldover *et al.* [6] display a spectrum measured in an argon-filled thick metal shell and we have measured resonances in a rigid plastic shell as well as in an inflatable plastic ball.

The resonant frequencies of a spherical loudspeaker whe-re 12 transducers are mounted on a spherical ABS plastic enclosure [2] were measured. The fundamental frequency f_{11} is very accurate and most other resonances match the the-ory fairly well.

The resonances of an inflatable plastic ball having a diameter of $0.67m$ were also measured by posing the plastic ball onto a small loudspeaker. The loudspeaker was playing test signals through the ball and a microphone recorded the sound filtered by the ball. The position of the microphone was chosen in order to balance the amplitude of the various resonances. Fig. 4 shows the frequency response measured with a white noise generator and a spectrum analyzer. We found that the low-frequency resonances are systematically sharper than the theoretical values. These deviations might be due to the compliance offered by the plastic boundary, which can not be considered as a rigid wall at low frequency. The prominent resonances could be identified with f_{11}, f_{21}, f_{31}, f_{41}, f_{51}, f_{61}, f_{71} and f_{91}.

5. SPHERICAL RESONATOR MODEL

Consider the perfectly-reflecting rectangular box and it-s representation in the BaBo model. Let us see how the model can be extended to spherical enclosures.

Indeed we would like to consider a perfectly reflecting sphere as a parallel of non-harmonic comb filters. The res-onances of the n^{th} comb filter correspond to the local ex-tremal points of the n^{th} order spherical Bessel function.

Given the resonance frequencies, we can sketch the ideal phase response of the n^{th} comb filter loop, since the loop phase has to be equal to a multiple of 2π in order to sustain the mode associated with a resonance. Fig. 1.a shows with crosses the phase response of the 0 order feedback loop at the resonance points. A monotonic phase curve interpo-lating those points can be obtained as the sum of a linear ramp and a nonlinear residual, also shown in fig. 1.a with dots and circles, respectively. The linear component is giv-en by a delay line whose length is equal to the slope of the linear ramp. The nonlinear residual can be provided by an allpass filter. The nonlinear phase curve can be roughly ap-proximated by a couple of linear segments, a low-frequency slope and a high-frequency slope. With this observation, the allpass filter for the 0 order Bessel function can be de-signed by placing the poles on the unit circle according to these two slopes. Fig. 1.b shows a zero-pole distribution that gives the two-slope phase response depicted in solid line in fig. 1.a.

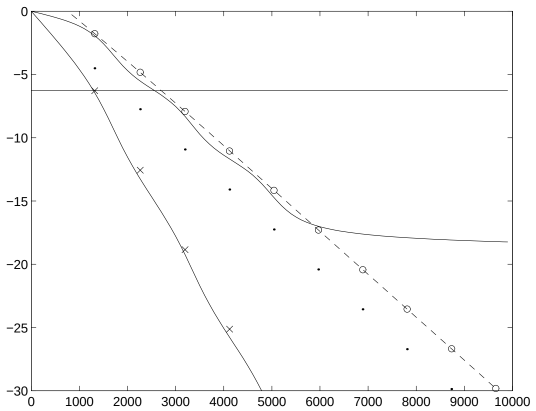
6. DESIGN PROCEDURE AND EXAMPLES

We have written a design procedure that finds the combi-nations of allpass filter and delay length for a set of sphere radii by iterative optimization over the position of the knee and the second, largest slope of the curve. In other word-s, the procedure iteratively changes the angle of the low-frequency pole and the relative, constant distance between all the other poles. The relative contribution of the delay line to the loop phase response is also subject to optimiza-tion. The distance of the poles from the center is kept fixed as it is mainly responsible for the magnitude of the ripples in the phase response.

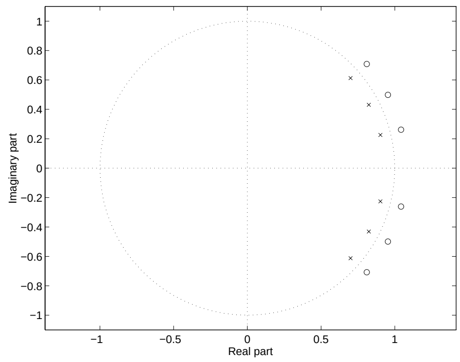
The design procedure can be run for several values of radius, in such a way to have a complete set of parameters for the resulting inharmonic comb filters. Notice that, even though a sixth-order allpass filter has six coefficients, our initial observation allows to implement it as three second-order sections that can be controlled by two parameters: the angle of the first pole, and the angular distance of the

[1]The spherical Bessel function of order 0 is just the popular sinc function $\sin x/x$.

[2]We thank Speaker Array Logic for providing the loudspeaker.



(a)



(b)

Figure 1: *(a): Phase response of the feedback loop of a inharmonic comb filter reproducing the resonances of a spherical resonator ($r = 0.188m$) associated with the Bessel function of order 0: $\times$: phase response at resonance points; $\cdot$: phase provided by the delay (8 samples); $\circ$: target phase residue to be approximated by the allpass filter; dashed line: polynomial curve approximating the target phase points; solid lines: designed allpass filter phase response and overall approximated phase response. (b): Pole-zero plot of the designed allpass filter.*

following poles. Since the precise position of resonances is of some importance only in the first few thousands Hz (say, 4kHz) [7], we see that a low order allpass filter is adequate for small spheres, e.g. order 6 or less for radii smaller than 0.5m. For larger spheres, the filter order should be increased in order to have a decent approximation at least under the first kHz.

Fig. 2 shows the frequency response of the parallel of dispersive comb filters here designed for radius 0.188. The crosses represent the ideal modal positions for Bessel functions of order 0 to 4. In order to have a good match between the first resonance of each comb and its theoretical position, we properly shaped a weighting function to be used in the iterative optimization procedure. Maximum weight is used around the first resonance, while the following resonances become gradually less important. Psychoacoustic investigations should be conducted in order to better understand if the approximations introduced can be perceived and if they affect the perceived object shape. However, informal listening seems to indicate that significant deviations from the theoretical partial positions can be tolerated without loosing the “sense of sphericity”.

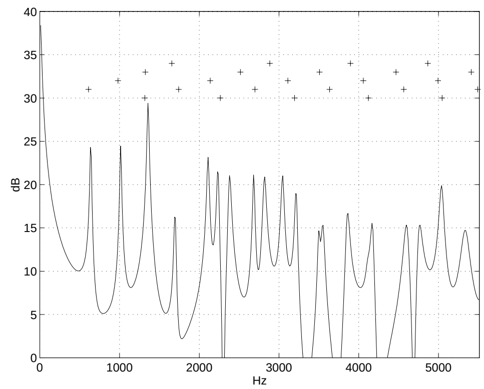


Figure 2: *Frequency response of the parallel of dispersive comb filters designed for radius 0.188. +: resonance positions of the ideal sphere up to Bessel series of order 4.*

Fig. 3 shows how the comb filters from order 0 to 3 separately contribute to the response of fig. 2, which is obtained by pure summation of the comb outputs. Around some resonances different modes coming from different Bessel series interact with each other, and the local result is either a magnification or an attenuation of the peak. As well as with actual enclosures where the shape of the frequency response is dependent on the positions of exciter and pickup, with the FDN we can vary the shape of the response, without moving the resonances, just by changing the input and output coefficients [4].

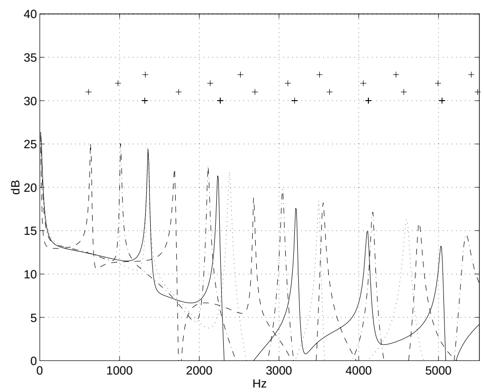


Figure 3: *Superimposition of the responses of the dispersive comb filters designed for radius 0.188. Solid line: Bessel order 0. Dashed line: Bessel order 1. Dash-dotted line: Bessel order 2. Dotted line: Bessel order 3.*

7. TUNING AND TRIALS

The FDN has been optimized according to the theory of the sphere, but, in order to compare it with the plastic ball, deviations from the theoretical tuning had to be implement-ed. Such deviations can be introduced in our model just by moving the theoretical resonance positions in the procedure for designing the allpass filters. This can be done fairly easily if the deviations are small, otherwise it can be difficult to assign a certain resonance to an inharmonic comb series. Alternatively, one can start with the filters designed for the ideal sphere and adjust the position of the first pole, as we did to obtain the frequency response of fig. 5, which should be compared with 4. In the feedback loop, we used second order FIR filters (exhibiting a one-sample delay) to simulate the faster attenuation of higher modes. Moreover, a first-order lowpass filter has been cascaded with the whole structure in order to resemble the lowpass characteristic of fig. 4.

In order to test the ability of the model to simulate actual objects, we have processed sounds through the FDN and have listened to the output and compared it with sounds that were recorded through the plastic ball.

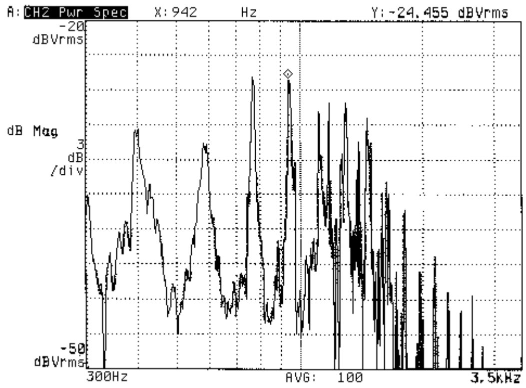


Figure 4: *Measured frequency response of the plastic ball.*

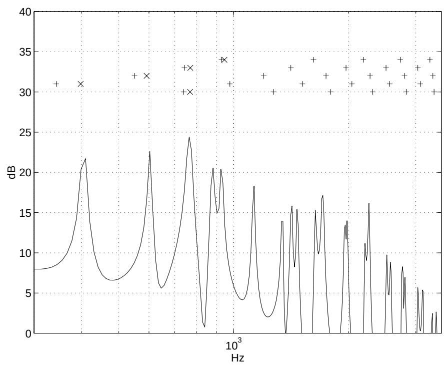


Figure 5: *Frequency response of the FDN model of the plastic ball.* $+$*: resonance positions of the ideal sphere.* $\times$*: measured resonance positions.*

8. CONCLUSION

We have demonstrated the feasibility of the extension of the BaBo model to spherical geometries. We have shown that a simple design procedure and, possibly, some manual tweaking, allow to realize an efficient structure that can be tuned either like an ideal sphere or like a real one. An open question is nevertheless how accurate this design has to be, since it is still unsettled which are the relevant parameters that affect the "perceived sphericity" of the object.

An important aspect of the model is that very few parameters are added to the BaBo model to control the allpass filters of the spherical model. Even deviations from ideality are reasonably achieved by moving only the position of one pole per inharmonic series. So far, we have simulated inharmonic series given by Bessel functions of order ranging from 0 to 6. In many cases, higher order inharmonic series are needed to achieve realism, but the fundamental resonance of those series seems to be most important while the higher resonances get drowned in the dense mixture of resonances from other series and their position is out of the bandwidth of perceived inharmonicity. So, we suggest to implement higher-order series as harmonic comb filters tuned (with some form of interpolation) to the fundamental frequency of that series.

If the inharmonic series, each corresponding to a Bessel function of a certain order, are recreated by comb filters having a delay line and an allpass filter in the feedback loop, it is conceivable to control the degree of "roundness" of the enclosure by changing the relative contribution to the overall phase response given by the delay and by the allpass filter. Namely, if all the delays are increased we gradually move from a sphere to a cube with rounded faces. This continuous shape control, as well as the extension of the BaBo model to cylindrical shapes will be covered in future research.

9. REFERENCES

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