EXTRACTION OF THE EXCITATION POINT LOCATION ON A STRING USING WEIGHTED LEAST-SQUARE ESTIMATION OF A COMB FILTER DELAY

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ABSTRACT

This paper focuses on the extraction of the excitation point location on a guitar string by an iterative estimation of the structural parameters of the spectral envelope. We propose a general method to estimate the plucking point location, working into two stages: starting from a measure related to the autocorrelation of the signal as a first approximation, a weighted least-square estimation is used to refine a FIR comb filter delay value to better fit the measured spectral envelope. This method is based on the fact that, in a simple digital physical model of a plucked-string instrument, the resonant modes translate into an all-pole structure while the initial conditions (a triangular shape for the string and a zero-velocity at all points) result in a FIR comb filter structure.

1. INTRODUCTION

Among the instrumental gesture parameters that contribute to the timbre of a guitar sound, the location of the plucking point along the string has a major influence. Plucking a string close to the bridge produces a tone that is softer in volume, brighter and sharper. The sound is richer in high-frequency components. This happens when playing the guitar *sul ponticello*. The other extreme is playing *sul tasto*, near or over the fingerboard, closer to the midpoint of the string. In that case, the tone is louder, rounder, mellower, less rich in high-frequency components.

2. PLUCKING A STRING AND COMB FILTERING

The plucking excitation initiates wave components travelling independently in opposite directions along the string. The resultant motion consists of two bends, one moving clockwise and the other counter-clockwise around a parallelogram [1]. In the ideal cases, the output from the string (force at the bridge) lacks those harmonics that have a node at the plucking point.

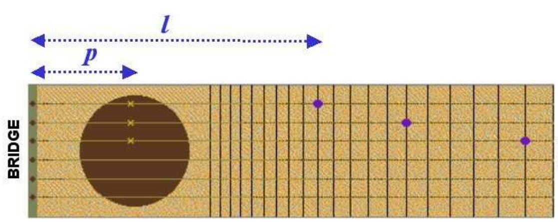

Figure 1: *Plucking point at a distance p from the bridge and fingering point at distance l from the bridge on a guitar neck.*

The amplitude $\hat{C}_n$ of the nth mode of the displacement of an ideal vibrating string of length l plucked at a distance p from the bridge with an initial vertical displacement h is given by :

$$\hat{C}_n(h, R) = \frac{2h}{n^2\pi^2 R(1-R)} \sin(n\pi R) \qquad (1)$$

where $R = p/l$ is the relative plucking position, defined as the fraction of the string length from the point where the string was plucked to the bridge [2]. $\hat{C}_n$ is considered here to be a model of the amplitude, hence the hat ($\hat{}$) while C_n represents *measured* values or *observed* values.

The digital signal processing interpretation of the physical phenomenon is the following: in a simple digital physical model of a plucked-string instrument, the resonant modes translate into an all-pole structure, while the initial conditions (a triangular shape for the string and a zero-velocity at all points) result in a FIR comb filter structure. At a sampling rate f_s, the magnitude of the frequency response of such a filter is given by

$$|H_d(e^{j\Omega})| = 2\sin(\Omega d/2) = 2\sin(\pi\, d\, f/f_s) \qquad (2)$$

where the delay d can be a non-integer number of samples. This delay corresponds to the time the wave needs to travel from the plucking point to the fixed end of the string (the bridge or the nut) and back ($2p$). As the fundamental period T_o corresponds to the time the wave needs to travel along a distance that is two times the vibrating length of the string ($2l$), we obtain the relation

$$\frac{D}{T_o} = \frac{2p}{2l} = R \qquad (3)$$

where $D = d/f_s$ is the delay expressed in seconds. This relationship between the comb filter delay D and the relative plucking position R is at the basis of the analogy between the physical model (Eq. 1) and its digital signal processing interpretation (Eq. 2). In fact, it is possible to verify that the arguments of the sine functions in equations 2 and 1 are equivalent:

$$\pi\, d\, f/f_s = \pi D f = \pi R T_o f = \pi R(f/f_o) = n\pi R \qquad (4)$$

The comb filtering effect is illustrated on Figure 2 for a recorded guitar tone plucked 12 cm away from the bridge on a 58 cm open A-string (fundamental frequency = 110 Hz). The relative plucking position R is approximately 1/5 (12 cm / 58 cm = 1.483). If it was exactly 1/5 and if the string was ideal, all harmonics with indices that are multiples of 5 would be completely missing.

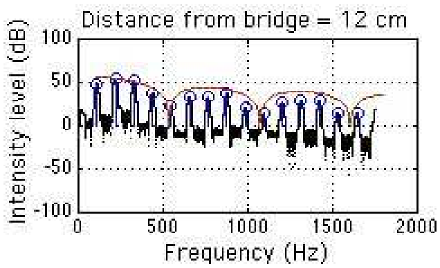

Figure 2: *Magnitude spectrum of a guitar tone, plucked at 12 cm from the bridge on a 58 cm string, showing the effect of the comb filtering with relative plucking position R close to 1/5.*

3. ESTIMATION OF COMB FILTER DELAY

A simple way to estimate the plucking point location along the string from a recording could be to look for the missing harmonics in the spectrum ($C_n = 0$). However, the string is usually not plucked exactly at a node of any of the lowest harmonics. That is why we propose in this paper a more general method to estimate the plucking point location, working into two stages: starting from a measure related to the autocorrelation of the signal as a first approximation, a weighted least-square estimation is used to refine the comb filter delay value to better fit the measured spectral envelope. This work builds on other methods proposed previously and reported in [3], [2] and [4], and can be easily extended to any situation that involves a comb filter.

3.1. First Approximation from Log-Correlation

The autocorrelation function $a(\tau)$ of a periodic signal $x(t)$ with fundamental period T_o can be expressed in terms of its Fourier series coefficients C_n in the following way:

$$a(\tau) = C_o^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2 \cos\left(\frac{2\pi}{T_o} n\tau\right) \quad (5)$$

While the long-term features of the autocorrelation function can be very useful to estimate the fundamental frequency of a periodic signal (since it should show a maximum at a lag corresponding to the fundamental period T_o), its short-term evolution can also tell us something about the plucking position.

Fig. 3 displays the plots of the autocorrelation function calculated for 12 recorded guitar tones plucked at various distances from the bridge on an open A-string (fundamental frequency = 110 Hz). As expected, the graphs show a maximum around $1/110 = 0.009$ seconds, the fundamental lag of the autocorrelation. One can also see that the autocorrelation takes on different shapes for different plucking positions but the information about the comb filter delay can not be extracted in an obvious way, directly from these graphs. As we want to detect the low amplitude harmonics, we modify the structure of the autocorrelation function by taking the log of the square of the Fourier coefficients (and by dropping the DC component). This emphasizes the contribution of low amplitude harmonics (around the *valleys* in the comb filter spectral envelope) by introducing large negative weighting coefficients. The obtained *log-correlation* is expressed as follows:

$$\Gamma(\tau) = \sum_{n=1}^{N} \log(C_n^2) \cos\left(\frac{2\pi}{T_o} n\tau\right) \quad (6)$$

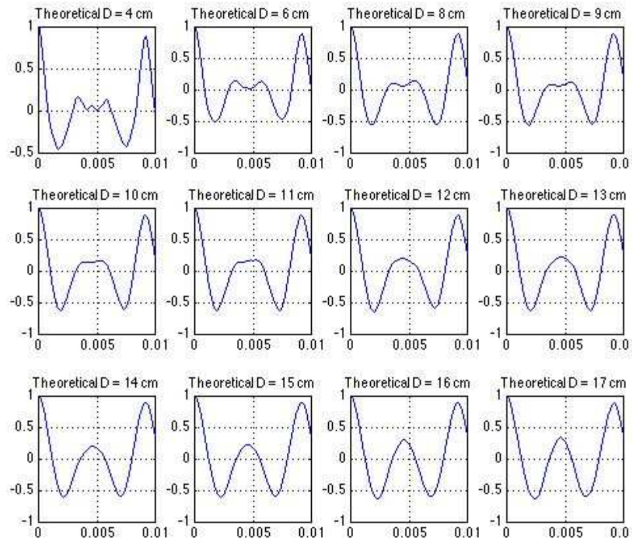

Figure 3: *Autocorrelation graphs for 12 guitar tones plucked at distances from the bridge ranging from 4 cm to 17 cm.*

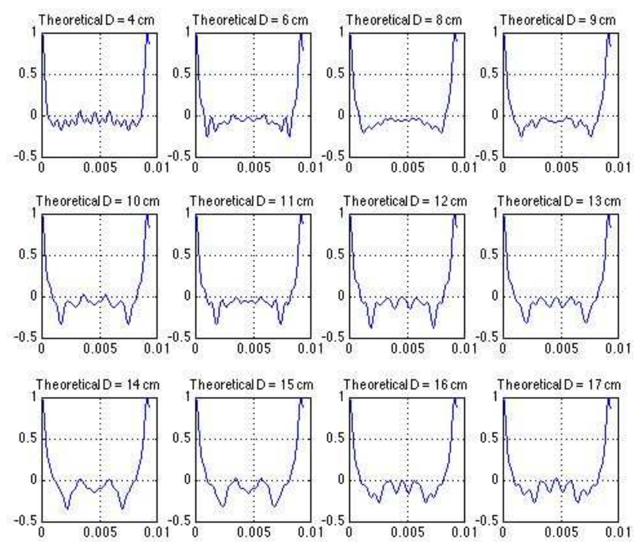

Figure 4: *Log-correlation graphs for 12 guitar tones plucked at distances from the bridge ranging from 4 cm to 17 cm.*

Fig. 4 displays the log-correlation graphs for the same 12 recorded guitar tones (as for Fig. 3). As expected, those plots reveal an interesting pattern: the minimum appears around the location of the lag corresponding to the plucking position. Therefore, we can conclude that the relative plucking position can be approximated by the ratio

$$R \approx \frac{\tau_{min}}{\tau_o} \quad (7)$$

where τ_{min} is the lag corresponding to the global minimum in the first half of the log-correlation period, and τ_o is the lag corresponding to the fundamental period T_o, as illustrated on Fig. 5.

A first approximation h_o for the vertical displacement h is also needed in order to initialize the weighted least-square procedure.

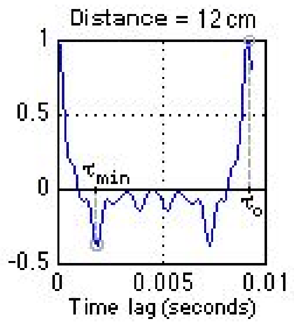

Figure 5: *Log-correlation for a guitar tone plucked 12 cm from the bridge on a 58 cm open A-string. Ratio* $\frac{\tau_{min}}{\tau_o}$ *provides a first approximation for relative plucking position R.*

h_o can be determined from the first approximation R_o of R and the total power of the harmonic components in the observed spectrum $\sum_{n \in I_W} C_n^2$,

$$h_o = R_o(1 - R_o)\frac{\pi}{2}\sqrt{\frac{\sum_{n \in I_W} C_n^2}{\sum_{n \in I_W} \frac{\sin^2(n\pi R_o)}{n^4}}} \tag{8}$$

I_W refers to the set of harmonics that are given a significant weight in the second stage of the approximation (as described in the next section).

3.2. Iterative Refinement of R Value using Weighted Least-Square Estimation

The second stage of the estimation consists in finding the values of h and R that minimize the distance between the theoretical expression of the ideal string magnitude spectrum $\hat{C}_n(h, R)$ (Eq. 1) and its observation $C_n(h, R)$. In fact, as illustrated on Fig. 6, we rather use the power coefficients C_n^2 for which it is not necessary to recover the phase (the phase of C_n being 0 or π).

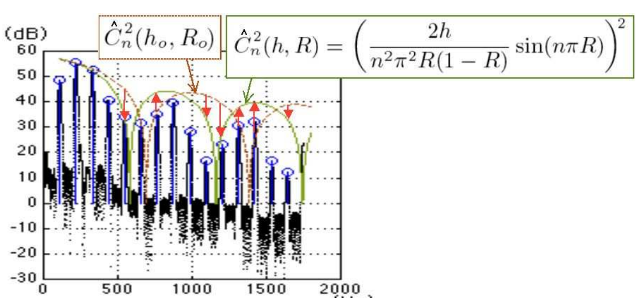

Figure 6: *Estimation of comb filter delay in two stages.*

$\hat{C}_n^2(h, R)$ is proportional to h^2 and $\sin^2(n\pi R)$ and is therefore a non linear expression in terms of h et R. A least-square estimation technique can still be used after linearizing $\hat{C}_n^2(h, R)$ with a first order Taylor's series approximation about a first approximation R_o of R and h_o of the height h of the string displacement. It leads to an expression of $\hat{C}_n^2(h, R)$ as a linear combination of the two correcting values $\Delta h = h - h_o$ and $\Delta R = R - R_o$ (omitting the $2/\pi^2$ factor):

$$\hat{C}_n^2(h, R) = \hat{C}_n^2(h_o, R_o) + \alpha_n \, \Delta h + \beta_n \, \Delta R \tag{9}$$

where

$$\begin{aligned} \hat{C}_n^2(h_o, R_o) &= \left(\frac{h_o \sin(n\pi R_o)}{n^2 R_o(1 - Ro)}\right)^2 \\ \alpha_n &= 2h_o\left(\frac{\sin(n\pi R_o)}{n^2 R_o(1 - Ro)}\right)^2 \\ \beta_n &= n\pi\left(\frac{h_o}{n^2 R_o(1 - R_o)}\right)^2 \sin(2n\pi R_o) \\ &\quad + \frac{2(2R_o - 1)}{R_o(1 - R_o)}\left(\frac{h_o \sin(n\pi R_o)}{n^2 R_o(1 - Ro)}\right)^2 \end{aligned}$$

In matrix form, Eq. 9 can be expressed as

$$[\hat{C}_n^2(h, R) - \hat{C}_n^2(h_o, R_o)] = \left[\begin{array}{cc} \alpha_n & \beta_n \end{array}\right] \cdot \left[\begin{array}{c} \Delta h \\ \Delta R \end{array}\right] \tag{10}$$

$$\hat{Y} = AX \tag{11}$$

where $\hat{Y}$ is the vector of differences between the estimated power spectrum and its first approximation. Since A is a $N \times 2$ matrix, the solution to Eq. 11 can be obtained using pseudo-inverse $(A^T A)^{-1} A^T$ or, for better results, its weighted version $(A^T W A)^{-1} A^T W$ where W is a $(N \times N)$ diagonal matrix containing the weights for the least-square errors. The weighting function can be used to select particular ranges of frequencies or reject components that are known for deviating from the theoretical comb filter model (near resonant frequencies of the guitar body for example). A good weighting curve is one that combines a bell curve and a positive sloped ramp. The bell curve increases the contribution of the components in the valleys of the spectrum and the ramp gives more weight to higher order and weaker harmonics over the whole range of the spectrum.

Finally, the correcting values for h and R are obtained with

$$\left[\begin{array}{c} \Delta h \\ \Delta R \end{array}\right] = [\,(A^T W A)^{-1} A^T W\,] \cdot [\,C_n^2 - \hat{C}_n^2(h_o, R_o)\,]$$

minimizing the distance between the model and the observation $||\hat{C}_n^2 - C_n^2||$ in a least-square sense. Then, the two parameters R and h are iteratively refined using $h_o + \Delta h$ and $R_o + \Delta R$ as second approximations and so on.

Between 3 to 10 iterations are generally needed to converge with a criterion error $\epsilon = \left|\frac{R_k - R_{k-1}}{R_k}\right| < 0.001$. As expected, the number of iterations decreases with the accuracy of the first approximation. If the first approximation is very rough ($\epsilon \simeq 0.5$), the number of iterations can increase to about 40 but the algorithm still converges to the right value of R (and h).

Fig. 7 displays the plots of the power spectrum of the 12 guitar tones together with the profile of the comb filter before and after iterative refinement. Fig. 8 displays the graph of the estimated plucking position $\hat{p}$ vs the actual distance from the bridge p in centimeters for the 12 guitar tones. The diagonal line indicates the target of the estimation (the actual value). The upper window displays a first approximation (obtained with log-correlation for example). The lower window shows the improvement achieved after refinement of R value using weigthed least-square estimation. For this data set, the average error is 0.78 cm for the first approximation and then is reduced to 0.18 cm after refinement.

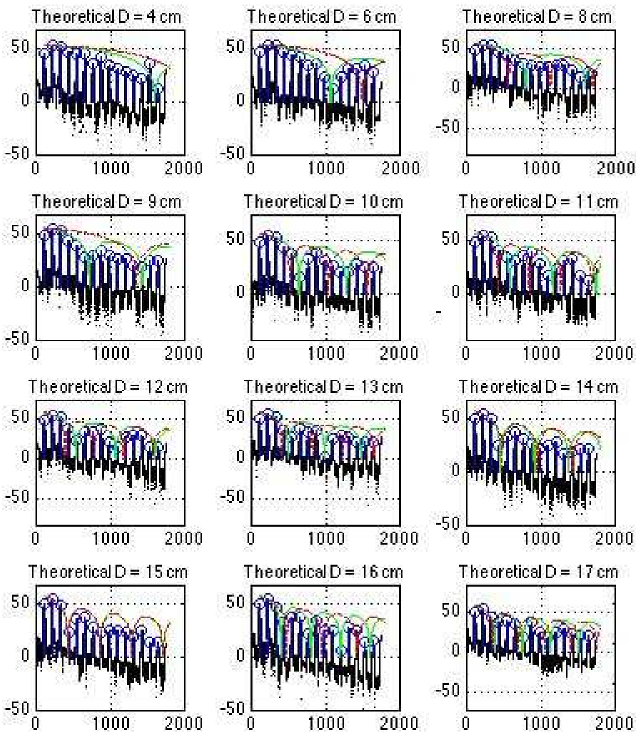

Figure 7: *Power spectra of 12 recorded guitar tones with superimposed comb filter model. First approximation plotted with a dark dashed line and final estimation plotted with a light gray line.*

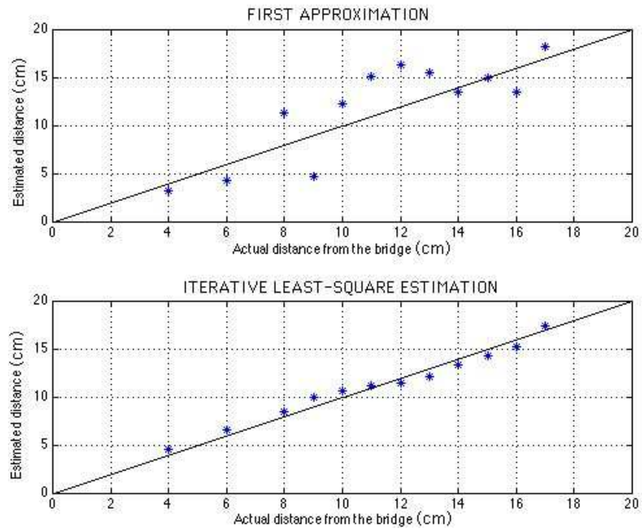

Figure 8: *Estimated vs actual plucking distances before (top window) and after (bottom window) refinement of* p *value using iterative weighted least-square estimation.*

4. CONCLUSION

This paper proposes an efficient method for the extraction of the excitation point location on a guitar string from a recording. It is based on the assumption that the power spectrum of a plucked string tone is comb-filter shaped. The plucking point location is estimated in two stages. Starting from a measure related to the autocorrelation of the signal as a first approximation, an iterative weighted least-square estimation is used to refine the comb filter delay value to better fit the measured spectral envelope.

Many applications can benefit from the algorithm, especially in the context of automatic tablature generation and sound synthesis (extraction of control parameters). This technique can also be used to derive the value of the delay of any kind of comb filter from the spectral peak parameters.

5. APPENDIX

The recorded tones that are used in this study were played with a plastic pick, 0.88 millimeters in thickness and triangular shaped, on a plywood classical guitar strung with nylon and nylon-wrapped steel Alvarez strings. The intended plucking locations were precisely measured and indicated on the string with a marker. The tones were recorded with a Shure KSM32 microphone in a sound-deadened room, onto digital audio tape at 44.1 kHz, 16 bits. The microphone was placed in front of the sound hole, approximately 25 cm away, which was far enough to capture a combination of waves coming from different parts of the string, in that way limiting the filtering effect of the pick-up point. A 4096-samples portion was extracted from the middle of the tone (after the attack) and the Fast Fourier Transform analysis was performed with zero-padding factor of 8 and parabolic interpolation. The magnitudes of the first 15 harmonics were used to calculate the log-correlation.

6. REFERENCES

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