DIGITAL SOUND SYNTHESIS OF BRASS INSTRUMENTS BY PHYSICAL MODELING

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ABSTRACT

The Functional Transformation Method (FTM) is an established method for sound synthesis by physical modeling, which has proven its feasibility so far by the application to strings and membranes. Based on integral transformations, it provides a discrete solution for continuous physical problems given in form of initial-boundary-value problems. This paper extends the range of applications of the FTM to brass instruments. A full continuous physical model of the instrument, consisting of an air column, a mouthpiece and the player's lips is introduced and solved in the discrete domain. It is shown, that the FTM is a suitable method also for sound synthesis of brass instruments.

1. INTRODUCTION

In the field of sound synthesis physical modeling has gained much interest in the last decade. The FTM steps in, where other physical based sound synthesis techniques loose direct physical relations or require high computational cost. The FTM starts with a description of a musical instrument in form of a partial differential equation (PDE) with initial conditions (IC) and boundary conditions (BC). By performing integral transformations on both time and space variable, a multi-dimensional transfer-function-model (MD TFM) is achieved, that allows the real time solution of the entire system. The FTM provides full access to all physical parameters of the underlying model. For that reason interaction of control parameters with the model is close to the control mechanisms of real musical instruments, which allows a intuitive way of interaction with the model. Furthermore fully accessible physical models can help to understand the mechanism of the sound production and therefore the influence and optimization of physical parameters. The general procedures of the FTM are depicted in Figure 1. Detailed information is available in [1].

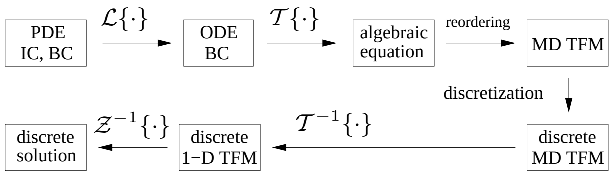

Figure 1: *General procedures of the FTM.*

In this paper a physical model of a brass instrument is built of several separate models: the air column, the mouthpiece and the player's lips. All models are chosen to be as complex as necessary and as simple as possible in order to keep the basic physical properties and to be able to show the feasibility of the FTM synthesis for brass instruments in general.

For the air column an extended version of Webster's horn equation is the underlying PDE of the model. The mouthpiece model is assumed to be a lumped model, where advantage is taken of analogous acoustic network circuits for the description of acoustical problems. This allows a description of the mouthpiece with a set of coupled ordinary differential equations (ODEs). Furthermore a lip model in form of a pressure controlled valve is used, that is coupled to the mouthpiece ODEs in a nonlinear way.

The paper is organized as follows. All parts of the entire model are treated in separate chapters. First they are introduced in the continuous form and then the way for obtaining a discrete model is shown respectively. Section 2 treats the air column and the used FTM procedures, Section 3 the mouthpiece model and Section 4 the lip model. Some details on the connection of the discrete models are given in Section 5. Section 6 provides results of the MATLAB implementation. Section 7 concludes this paper and shows possibilities of improvement of the physical model.

2. AIR COLUMN

Brass instruments consist roughly of long cylindrical pipes, mostly curved, that flare to a horn at one end. The sound waves inside propagate lossy through the instrument and are both, radiated and reflected at the horn. Due to viscous and thermal effects at the walls of the instrument and high pressure amplitudes the significant loss mechanisms are quite complex. Details are available in [2]. However in that first approach towards brass instruments the model is kept quite simple.

2.1. Continuous Model

Starting point of the air column model is a mathematical description of the brass instrument as an initial-boundary-value problem. Therefore, corresponding to [1], the following description with a PDE in vector notation with initial values $\mathbf{y}_i$ and boundary values $\mathbf{y}_b$ is used.

$$\begin{aligned} [\mathbf{L} + \mathbf{C}D_\mathrm{t}]\,\mathbf{y}(\mathbf{x},t) &= \mathbf{f}_\mathrm{e}(\mathbf{x},t), \quad \mathbf{x} \in V \\ \mathbf{f}_\mathrm{b}^H \mathbf{y}(\mathbf{x},t) &= \mathbf{y}_\mathrm{b}(\mathbf{x},t), \quad \mathbf{x} \in \partial V \\ \mathbf{f}_\mathrm{i}^H \mathbf{y}(\mathbf{x},t)|_{t=0} &= \mathbf{y}_\mathrm{i}(\mathbf{x}), \quad \mathbf{x} \in V \end{aligned} \tag{1}$$

The vector of unknown quantities is $\mathbf{y}(\mathbf{x},t)$. The vector $\mathbf{f}_\mathrm{e}(\mathbf{x},t)$ indicates the excitation functions. The variable t is the time. $\mathbf{x}$ denotes the vector of spatial coordinates defined in the spatial volume V, which is bounded by ∂V. The operator D_t denotes the 1st order temporal derivative. $\mathbf{f}_\mathrm{i}$ and $\mathbf{f}_\mathrm{b}$ are vector operators specifying initial and boundary values. $(\cdot)^H$ is the hermitian operation and $\mathbf{L}$ is a matrix operator of the form

$$\mathbf{L} = \mathbf{A} + \mathbf{B}\nabla \ . \tag{2}$$

At first the model is simplified to one spatial dimension. In consequence the nabla operator ∇ simplifies to the 1st order derivative with respect to the spatial coordinate x. The model divides the spatial volume V of the brass instrument model into a cylindrical part V_1 and an horn shaped part V_2, which are both assumed to be round in diameter.

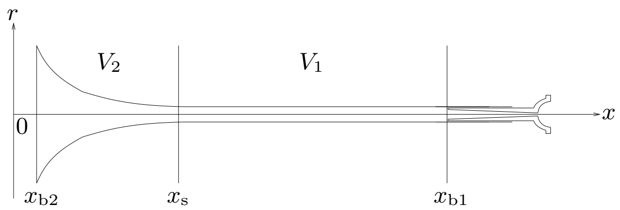

Figure 2: *Profile of a brass instrument. Details of the shape vary with the instrument. At the coordinate x_{b1} the air column is connected to the mouthpiece. x_{b2} is the coordinate of the bell's end. The cylindrical and the horn shaped part are connected at x_{s} .*

For a concise notation the velocity potential Φ is introduced, that is related to the pressure p and the sound particle velocity v by

$$\frac{\partial \Phi(x,t)}{\partial x} = \Phi'(x,t) = v(x,t) \ , \tag{3}$$
$$\frac{\partial \Phi(x,t)}{\partial t} = \dot{\Phi}(x,t) = -\frac{1}{\varrho_0}\ p(x,t) \ . \tag{4}$$

The constant ϱ_0 is the static density of air. The vector $\mathbf{y}$ of unknown quantities for the initial-boundary-value problem is

$$\mathbf{y}(x,t) = \begin{pmatrix} \Phi'(x,t) \\ \dot{\Phi}(x,t) \end{pmatrix} \ . \tag{5}$$

It contains the flux quantity in form of Φ' and the potential quantity in form of $\dot{\Phi}$. The standard PDE used for the air column model is the well known horn equation of Webster. Based on known eigenfunctions for hyperbolic horns, it has been already investigated in [3]. Here, we consider an extended version of Webster's horn equation with an additional term $d_1\dot{\Phi}$, that causes a frequency-independent damping effect in the air column model. This is a severe simplification, as the damping in real brass instruments is frequency-dependent. Details are available in [2]. The resulting equation is valid for the sound particle velocity potential Φ as well as for the pressure p

$$\Phi'' + \frac{A'(x)}{A(x)}\Phi' = \frac{1}{c^2}\ddot{\Phi} + d_1\dot{\Phi} \ . \tag{6}$$

The function $A(x)$ is the diameter of the wavefront within the instrument dependent on the coordinate x. $A'(x)$ denotes the the 1st order derivative of $A(x)$ with respect to the variable x. The constant c is the speed of sound in air.

The scalar PDE of equation (6) is turned into a vector PDE, corresponding to the notation in equation (1), with the following matrices $\mathbf{A}$ and $\mathbf{C}$.

$$\mathbf{A} = \begin{pmatrix} \frac{A'(x)}{A(x)} & -d_1 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 & -\frac{1}{c^2} \\ -1 & 0 \end{pmatrix} \tag{7}$$

The matrix $\mathbf{B}$ equals the identity matrix $\mathbf{I}_0$. In the cylindrical part of the instrument the radius of the instrument's pipe is constant. Thus we have a constant radius of r_c with

$$r(x) = r_{\mathrm{c}}, \qquad \frac{A'(x)}{A(x)} = 0, \qquad x \in V_1 \ . \tag{8}$$

For the horn shaped part we assume that the shape of a brass instrument's horn is close to a Bessel horn. The radius function r of such a horn is given with

$$r(x) = r_{\mathrm{h}} x^{-\varepsilon}, \qquad \frac{A'(x)}{A(x)} = -\frac{2\varepsilon}{x}, \qquad x \in V_2 \ , \tag{9}$$

where ε is the flare parameter of the horn and r_h is a scaling factor. A brass instrument's shape is achieved by adjusting the parameters x_{b1}, x_{b2}, x_{s}, r_{c}, r_{h} and ε properly. The excitation forces in V are set to zero and the initial conditions are assumed to be homogeneous.

$$\mathbf{f}_{\mathrm{e}}(x,t) = \mathbf{0}, \tag{10}$$
$$\mathbf{y}_{\mathrm{i}}(x) = 0 \ . \tag{11}$$

The boundary conditions have to be adapted to our problem. At the point x_{b1}, where the mouthpiece is connected to the air column, we assume the sound particle velocity being equal to a given boundary excitation function $\psi(t)$. At the boundary point x_{b2} a radiation load can model sound radiation from the instrument. To keep the model simple, the radiation load is set to zero, which means the pressure is zero at x_{b2}. Thus the boundary conditions are determined as follows:

$$\mathbf{f}_{\mathrm{b1}}^H \mathbf{y}(x_{\mathrm{b1}},t) = \psi(t) \ , \tag{12}$$
$$\mathbf{f}_{\mathrm{b2}}^H \mathbf{y}(x_{\mathrm{b2}},t) = 0 \ , \tag{13}$$

$$\mathbf{f}_{b1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{f}_{b2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \ . \tag{14}$$

2.2. Discrete Model

A discrete solution of the continuous problem can be obtained by performing the steps of FTM procedure as shown in Figure 1. Performing the Laplace-transformation on the problem of equation (1) leads to the following ODE with boundary conditions.

$$[\mathbf{L} + s\mathbf{C}]\,\mathbf{Y}(x,s) = \mathbf{0} \qquad x \in V \tag{15}$$
$$\mathbf{f}_{b1}^H \mathbf{Y}(x_{b1},s) = \Psi(s) \tag{16}$$
$$\mathbf{f}_{b2}^H \mathbf{Y}(x_{b2},s) = 0 \tag{17}$$

Next step of the FTM is the Sturm-Liouville-Transformation (SLT) that is used for the transformation of the space variable. The SLT is an integral transformation with a kernel function $\tilde{\mathbf{K}}(x,\tilde{\beta})$ that has to be designed carefully in order to obtain a transfer function model from the problem (see [1]). The transformation is defined by

$$\mathcal{T}\{\mathbf{Y}(x)\} = \bar{Y}(\tilde{\beta}) = \int_V \tilde{\mathbf{K}}^H(x,\tilde{\beta})\mathbf{C}\mathbf{Y}(x)dx \ . \tag{18}$$

Using a well designed kernel function transformation of equation (15) yields an algebraic equation in the transformed domain that includes the transformed boundary values $\bar{Y}_b$ as follows:

$$\beta\bar{Y}(\tilde{\beta},s) + s\bar{Y}(\tilde{\beta},s) - \bar{Y}_{\mathrm{b}}(\tilde{\beta},s) = 0 \ . \tag{19}$$

In order to obtain an algebraic equation corresponding to (19) from equation (15) a set of Sturm-Liouville eigenvalue problems has to be solved. These problems are called the eigenvalue problem and the adjoint eigenvalue problem. Details on this step are left out here and can be found in [1]. Firstly the adjoint operator $\tilde{\mathbf{L}}$ is introduced.

$$\tilde{\mathbf{L}} = \mathbf{A}^H - \mathbf{B}^H \nabla \tag{20}$$

Because the model divides the instrument in two parts described with different PDEs due to the different matrix $\mathbf{A}$ in V_1 and V_2, the eigenvalue problem and the adjoint eigenvalue problem have to be solved in sections for the definition range V_1 and V_2 respectively. The kernel functions $\mathbf{K}$ and $\tilde{\mathbf{K}}$ are then defined section-wise (see [4] for details). The problems are indexed with $n = 1, 2$.

$$\mathbf{L}\mathbf{K}_n(x, \beta) = \beta \mathbf{C}\mathbf{K}_n(x, \beta), \quad x \in V_n \tag{21}$$

$$\mathbf{f}_{\mathrm{b}n}^H \mathbf{K}_n(x_{\mathrm{b}n}, \beta) = 0 \tag{22}$$

$$\mathbf{K}_1(x_{\mathrm{s}}, \beta) = \mathbf{K}_2(x_{\mathrm{s}}, \beta) \tag{23}$$

$$\tilde{\mathbf{L}}\tilde{\mathbf{K}}_n(x, \tilde{\beta}) = \tilde{\beta} \mathbf{C}^H \tilde{\mathbf{K}}_n(x, \tilde{\beta}), \quad x \in V_n \tag{24}$$

$$\tilde{\mathbf{f}}_{\mathrm{b}n}^H \tilde{\mathbf{K}}_n(x_{\mathrm{b}n}, \tilde{\beta}) = 0 \tag{25}$$

$$\tilde{\mathbf{K}}_1^H(x_{\mathrm{s}}, \tilde{\beta}) = \tilde{\mathbf{K}}_2^H(x_{\mathrm{s}}, \tilde{\beta}) \tag{26}$$

The adjoint boundary operators $\tilde{\mathbf{f}}_{\mathrm{b}1}$ and $\tilde{\mathbf{f}}_{\mathrm{b}2}$ are

$$\tilde{\mathbf{f}}_{\mathrm{b}1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \tilde{\mathbf{f}}_{\mathrm{b}2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{27}$$

For concise notation of the solution of (21) and (24) we introduce $\nu = \varepsilon + \frac{1}{2}$, $M = \frac{\sqrt{\beta^2 - \beta d_1 c^2}}{c}$ and $\tilde{M} = \frac{\sqrt{\tilde{\beta}^2 - \tilde{\beta} d_1 c^2}}{c}$. J_n and Y_n denote the Bessel-Functions of order n of 1st and 2nd kind. The following solutions can be obtained by any symbolic mathematical program as MAPLE, for instance. The solution of equation (21) is

$$\mathbf{K}_1(x, \beta) = \begin{pmatrix} -\frac{M}{\beta} e^{Mx} & \frac{M}{\beta} e^{-Mx} \\ e^{Mx} & e^{-Mx} \end{pmatrix} \mathbf{C}_K^{(1)} \tag{28}$$

$$\mathbf{K}_2(x, \beta) = x^{\nu} \begin{pmatrix} \frac{jM}{\beta} J_{1-\nu}(jMx) & \frac{jM}{\beta} Y_{1-\nu}(jMx) \\ J_{-\nu}(jMx) & Y_{-\nu}(jMx) \end{pmatrix} \mathbf{C}_K^{(2)} \tag{29}$$

$$\mathbf{C}_K^{(1)} = \begin{bmatrix} e^{-Mx_{\mathrm{b}1}} \\ e^{Mx_{\mathrm{b}1}} \end{bmatrix} a_K^{(1)}, \quad \mathbf{C}_K^{(2)} = \begin{bmatrix} Y_{-\nu}(jMx_{\mathrm{b}2}) \\ -J_{-\nu}(jMx_{\mathrm{b}2}) \end{bmatrix} a_K^{(2)} \tag{30}$$

with $a_K^{(1)}$ and $a_K^{(2)}$ being constants. The solution of equation (24) is

$$\tilde{\mathbf{K}}_1(x, \tilde{\beta}) = \begin{pmatrix} e^{\tilde{M}x} & e^{-\tilde{M}x} \\ \frac{\tilde{M}}{\tilde{\beta}} e^{\tilde{M}x} & -\frac{\tilde{M}}{\tilde{\beta}} e^{-\tilde{M}x} \end{pmatrix} \tilde{\mathbf{C}}_K^{(1)} \tag{31}$$

$$\tilde{\mathbf{K}}_2(x, \tilde{\beta}) = x^{1-\nu} \begin{pmatrix} \frac{\tilde{\beta}}{j\tilde{M}} J_{\nu}(j\tilde{M}x) & \frac{\tilde{\beta}}{j\tilde{M}} Y_{\nu}(j\tilde{M}x) \\ J_{\nu-1}(j\tilde{M}x) & Y_{\nu-1}(j\tilde{M}x) \end{pmatrix} \tilde{\mathbf{C}}_K^{(2)} \tag{32}$$

$$\tilde{\mathbf{C}}_K^{(1)} = \begin{bmatrix} e^{-\tilde{M}x_{\mathrm{b}1}} \\ e^{\tilde{M}x_{\mathrm{b}1}} \end{bmatrix} \tilde{a}_K^{(1)}, \quad \tilde{\mathbf{C}}_K^{(2)} = \begin{bmatrix} Y_{\nu}(j\tilde{M}x_{\mathrm{b}2}) \\ -J_{\nu}(j\tilde{M}x_{\mathrm{b}2}) \end{bmatrix} \tilde{a}_K^{(2)} \tag{33}$$

with $\tilde{a}_K^{(1)}$ and $\tilde{a}_K^{(2)}$ being constants. Then the eigenvalues β_μ and the adjoint eigenvalues $\tilde{\beta}_\mu$ can be obtained from equation (23) and (26) respectively. Inserting the kernels (28) and (29) in equation (23) leads to the following complex equation:

$$j \tanh\big(M(x_{\mathrm{s}} - x_{\mathrm{b}1})\big) = \frac{\Big(Y_{-\nu}(jMx_{\mathrm{b}1}) J_{1-\nu}(jMx_{\mathrm{s}}) - J_{-\nu}(jMx_{\mathrm{b}1}) Y_{1-\nu}(jMx_{\mathrm{s}})\Big)}{\Big(Y_{-\nu}(jMx_{\mathrm{b}1}) J_{-\nu}(jMx_{\mathrm{s}}) - J_{-\nu}(jMx_{\mathrm{b}1}) Y_{-\nu}(jMx_{\mathrm{s}})\Big)}. \tag{34}$$

In order to avoid a numerical search for the eigenvalues in the entire complex plane, we can take advantage of the frequency-independent damping effects. It is known a priori that this kind of damping gives the same real part for all eigenvalues β_μ. Under the assumption $\beta_\mu = \sigma + j\omega_\mu = \frac{d_1 c^2}{2} + j\omega_\mu$ the term M simplifies to $M = j\frac{\sqrt{\sigma^2 + \omega_\mu^2}}{c}$. With this assumption equation (34) simplifies to a real equation depending only on the variable ω_μ. Solutions of this simplified equation can be found numerically in a much easier way. The eigenvalues are related to the adjoint eigenvalues with $\beta_\mu^* = \tilde{\beta}_\mu$. The operation $(\cdot)^*$ denotes the conjugate complex of $(\cdot)$.

Application of the SLT provides the transformed boundary conditions $\bar{Y}_{\mathrm{b}}$. Details on this step are available in [1].

$$\bar{Y}_{\mathrm{b}}(\tilde{\beta}_\mu, s) = -\tilde{\mathbf{g}}_{\mathrm{b}1}^H \tilde{\mathbf{K}}(x_{\mathrm{b}1}, \tilde{\beta}_\mu) \Psi(s) \tag{35}$$

The operator $\tilde{\mathbf{g}}_{\mathrm{b}1}^H$ is a vector operator of the form $\begin{bmatrix} -1 & 0 \end{bmatrix}$. Reordering equation (19) and discretization by using the impulse-invariant-transformation yields a discrete-time transfer function model (T is the sampling interval).

$$\bar{Y}(\tilde{\beta}_\mu, z) = T \frac{z}{z - e^{-\tilde{\beta}_\mu T}} \bar{Y}_{\mathrm{b}}(\tilde{\beta}_\mu, z) \tag{36}$$

The boundary excitation function ψ has the quantity of a sound particle velocity and is connected to the excitation volume flow u_{e} at $x = x_{b1}$ by $\psi(t) = \frac{u_{\mathrm{e}}(t)}{A(x_{b1})}$. Thus we can rewrite equation (36) using equation (35) to

$$\begin{aligned} \bar{Y}(\tilde{\beta}_\mu, z) \;=\; & \bar{Y}(\tilde{\beta}_\mu, z) \underbrace{e^{-\tilde{\beta}_\mu T}}_{\zeta[\mu]} z^{-1} \\ & \underbrace{-\frac{T}{A(x_{\mathrm{b}1})} \tilde{\mathbf{g}}_{\mathrm{b}1}^H \tilde{\mathbf{K}}(x_{b1}, \tilde{\beta}_\mu)}_{\xi[\mu]} U_{\mathrm{e}}(z)\ . \end{aligned} \tag{37}$$

The vector of unknown quantities $\mathbf{y}$ can be obtained by performing the inverse SLT. Now it it possible to compute the pressure and the sound particle velocity of the air column for all values of x.

$$\mathbf{Y}(x, z) = \mathcal{T}^{-1}\{\bar{Y}(\tilde{\beta}_\mu, z)\} = \sum_\mu \underbrace{\frac{\mathbf{K}(x, \tilde{\beta}_\mu)}{N_\mu}}_{\kappa[\mu, x]} \bar{Y}(\tilde{\beta}_\mu, z) \tag{38}$$

In order to avoid aliasing the summation is limited up to the eigenvalues β_μ that fulfill the condition $\Im\{\beta_\mu\} < \frac{\pi}{T}$. The norm factor N_μ computes with

$$N_\mu = \int_{x_{b2}}^{x_{b1}} \tilde{\mathbf{K}}^H(x, \tilde{\beta}_\mu) \mathbf{C}\mathbf{K}(x, \beta_\mu) dx\ . \tag{39}$$

3. MOUTHPIECE

The mouthpiece is the connection between the lips of the player and the air column. There are brass mouthpieces in various shapes and sizes. The details vary with the style of the instrument and the player's preferences, but all mouthpieces have the common general design shown in Figure 3. The player presses the lips against the surface of the mouthpiece cup, which has a characteristic volume V_b. A narrower passage of the diameter S_c and a length of l_c connects the cup to the main bore of the instrument.

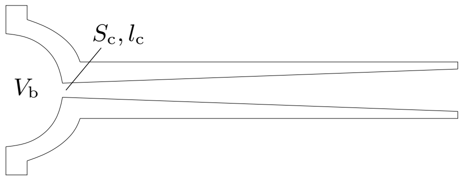

Figure 3: *Profile of a typical brass instrument mouthpiece*

3.1. Continuous Model

A simple physical model of a mouthpiece can be found in [2]. The analogous acoustic network depicted in Figure 4 is used to describe the basic physical behavior of a brass mouthpiece. The cup is modeled as an acoustic compliance C and the passage of constriction as an acoustic inertance L. Lossy effects in the mouthpiece are included in the model with an dissipative element R. The impedance quantities denote the ratio of the pressure p to the volume flow u. For a common linear network like the mouthpiece model the underlying ODEs are simple. Therefore a description of the mouthpiece model is given directly in the frequency domain with the following 2×2 input-output-system matrix

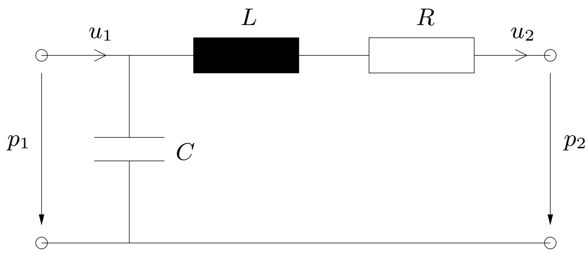

Figure 4: *Analogous acoustic network for a brass mouthpiece.*

$$\begin{pmatrix} U_1(s) \\ P_1(s) \end{pmatrix} = \begin{pmatrix} s^2LC + sRC + 1 & sC \\ sL + R & 1 \end{pmatrix} \begin{pmatrix} U_2(s) \\ P_2(s) \end{pmatrix} . \quad (40)$$

The values L and C of the physical model can be computed from the mouthpiece geometry.

$$C = \frac{V_\mathrm{b}}{\varrho_0 c^2} \quad (41)$$

$$L = \frac{\varrho_0 l_\mathrm{c}}{S_\mathrm{c}} \quad (42)$$

We assume as excitation a volume flow u_1 entering the mouthpiece. It is required to compute the flow out of the mouthpiece, denoted by u_2, from the flow u_1, to be able to determine the excitation function ψ for the air column. The air column behaves as a load coupled to port 2 of the mouthpiece model. The pressure p_1 corresponds to the output of the instrument's impedance model given by the mouthpiece and the coupled air column.

3.2. Discrete Model

A discrete mouthpiece model can be obtained by performing the impulse-invariant-transformation on the matrix description in equation (40). In the z-domain we get for U_2 and P_1

$$\begin{aligned} U_2(z) &= \left[a_1 z^{-1} + a_2 z^{-2}\right] U_2(z) + a_3 z^{-1} U_1(z) \quad (43) \\ &\quad + \left[a_4 + a_5 z^{-1}\right] P_2(z), \\ P_1(z) &= \left[a_6 + a_7 z^{-1}\right] U_2(z) + a_8 P_2(z) . \quad (44) \end{aligned}$$

The coefficients $a_1, ..., a_8$ can be obtained from the discretization procedure.

4. LIPS

So far a model for a brass instrument has been introduced. Beside the properties of the instrument itself the interaction mechanism of the player with the instrument has a strong influence on the produced sound and is fundamental for the characteristic sound of every instrument. When playing a brass instrument the player interacts with the instrument by using his lips, that are pressed against the mouthpiece. Thereby the lips behave as an oscillator, that excites the air column inside the instrument. The lip oscillation is supported with energy provided by the player himself and reflections coming back from the instrument. The lip model is chosen to provide the behavior of lips towards a brass instrument in general and neglects details.

4.1. Continuous Model

Physical lip models of different kinds are available from the publications of N.H. Fletcher, e.g. [5], and from various publications of the group of X.Rodet at IRCAM, Paris, e.g. [6]. All these lip models are pressure controlled valve models. Here a basic upward striking model is used.

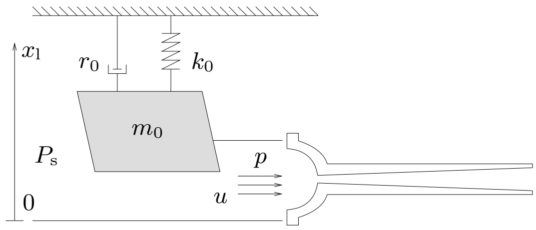

Figure 5: *Schematic of a physical model for the lips of a brass player according to [6].*

A mathematical description of the model is given by the following ODE.

$$m_0 \ddot{x}_1(t) + r_0 \dot{x}_1(t) + k_0 x_1(t) = \gamma(P_\mathrm{s} - p(t)) \quad (45)$$

Thereby x_1 is the opening of the lip, m_0 the mass of the lip, r_0 a damping constant and k_0 the spring constant. γ is a constant related to the geometric details of the lip model, P_s indicates the blowing pressure inside the mouth and p is the pressure inside the mouthpiece. The volume flow u entering the mouthpiece is set to zero except when the condition $x_1(t) > 0 \wedge P_\mathrm{s} - p(t) > 0$ is fulfilled. Then it is computed with

$$u(t) = l\sqrt{\frac{2}{\varrho_0}}x_1(t)\sqrt{P_\mathrm{s} - p(t)}\ . \tag{46}$$

The parameter l is a constant describing the width of the lip. The initial conditions are denoted with $\mathbf{x}_\mathrm{i} = \left[x_1(0)\ \ \dot{x}_1(0)\right]$.

4.2. Discrete Model

A discrete lip model can be easily obtained by performing the Laplace-transformation and the impulse-invariant-transformation on equation (45)

$$\begin{aligned} X_1(z) &= \left[w_1 z^{-1} + w_2 z^{-2}\right] X_1(z) + w_3 z^{-1}(P_\mathrm{s} - P(z)) \\ &\quad + \mathbf{x}_\mathrm{i}\mathbf{w}_0(z)\ . \end{aligned} \tag{47}$$

The coefficients w_1, w_2, w_3 and the vector $\mathbf{w}_0(z)$ are available from the discretization procedure.

5. CONNECTING THE MODELS

When the mouthpiece model is connected to the air column, the volume flow u_2 equals the excitation flow u_e of the air column model. Furthermore the pressure p_2 equals the pressure at the coordinate x_b1 of the air column. The lip model can be connected to the mouthpiece by setting the lip flow u equal u_1 and the mouthpiece pressure p equal p_1. But this straight forward approach of joining the models yields an algorithm containing a delay free loop. In order to get an implementable synthesis algorithm all delay free loops have to be eliminated. Computing u_2 with equation (43) does require p_2 to be known at the recent time step. But with equation (38) the state vector of the FTM, and consequently p_2, can only be computed when all FTM computations (equation (37)) of the recent time step are already performed. This would require knowledge about the still unknown flow u_2. Therefore this straight forward way is not possible. An successful way of joining the models is shown now. The output pressure p_2 can be obtained from the state vector of the FTM.

$$p_2(t) = -\varrho_0 \underbrace{\begin{pmatrix} 0 & 1 \end{pmatrix}}_{\mathbf{h}_\mathrm{p}} \mathbf{y}(x_\mathrm{b1}, t) \tag{48}$$

Using equation (37), (38) and (48) yields the following expression for p_2:

$$\begin{aligned} P_2(z) &= \underbrace{-\varrho_0 \mathbf{h}_\mathrm{p} \sum_\mu \kappa[\mu, x_\mathrm{b1}]\zeta[\mu]\bar{Y}(\tilde{\beta}_\mu, z)\, z^{-1}}_{\eta(z)} \\ &\quad - \varrho_0 \mathbf{h}_\mathrm{p} \sum_\mu \kappa[\mu, x_\mathrm{b1}]\xi[\mu] U_2(z)\ . \end{aligned} \tag{49}$$

Equation (49) can now be inserted into equation (43) and (44). Reordering yields the following equations with the coefficients $e_1..e_8$, that are available from the reordering procedure

$$\begin{aligned} U_2(z) &= \left[e_1 z^{-1} + e_2 z^{-2}\right] U_2(z) + e_3 z^{-1} U_1(z) \\ &\quad + \left[e_4 z^{-1} + e_5 z^{-2}\right]\eta(z) \end{aligned} \tag{50}$$

$$P_1(z) = \left[e_6 + e_7 z^{-1}\right] U_2(z) + e_8 z^{-1}\eta(z)\ . \tag{51}$$

With equation (50) u_2 can be computed without the knowledge of p_2, which means the delay free loop has been eliminated. So the following algorithm provides a discrete solution of the previously introduced continuous model. All equations given in the z-domain are transformed to the discrete time domain by performing the inverse z-transformation:

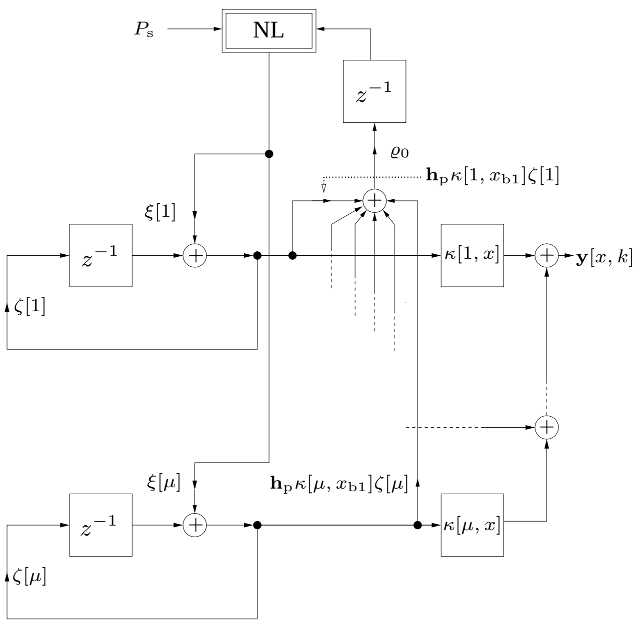

Figure 6: *Synthesis algorithm for brass instruments. Simulation of the air column is performed with several complex 1st-order resonators in parallel. The excitation is computed with a nonlinear loop back containing mouthpiece and lip model computations.*

for $k \in$ simulation time

$$\begin{aligned} u_2[k] &= e_1 u_2[k-1] + e_2 u_2[k-2] + e_3 u_1[k-1] \\ &\quad + e_4\eta[k-1] + e_5\eta[k-2] \end{aligned}$$

for $\mu = 1 : N$

$$\bar{y}[\mu, k] = \zeta[\mu]\bar{y}[\mu, k-1] + \xi[\mu]u_2[k]$$

$$\eta[k] = \sum_\mu \varrho_0 \mathbf{h}_\mathrm{p}\kappa[\mu, x_\mathrm{b1}]\zeta[\mu]\bar{y}[\mu, k]$$

$$\begin{aligned} x_1[k] &= w_1 x_1[k-1] + w_2 x_1[k-2] \\ &\quad + w_3\left(P_\mathrm{s}[k-1] - p[k-1]\right) \end{aligned}$$

$$p_1[k] = e_6 u_2[k] + e_7 u_2[k-1] + e_8\eta[k-1]$$

if $\quad x_1[k] > 0 \quad \wedge \quad P_\mathrm{s}[k] - p_1[k] > 0$

$$u_1[k] = l\sqrt{\frac{2}{\varrho_0}}x_1[k]\sqrt{P_\mathrm{s}[k] - p_1[k]}$$

else

$$u_1[k] = 0$$

6. RESULTS

The synthesis algorithm was implemented in MATLAB. The model parameters were adjusted to fit the profile of a trumpet and a trumpet player. At first the instrument model without joined lips was investigated by computing the impulse response of the system. This allowed comparisons of the physical modeled computer instrument to measurements performed on real instruments. Comparable data is available in [6]. The results showed conformity at least in the basic matters, which is reasonable when looking at the rough simplifications of the underlying physical model. In the

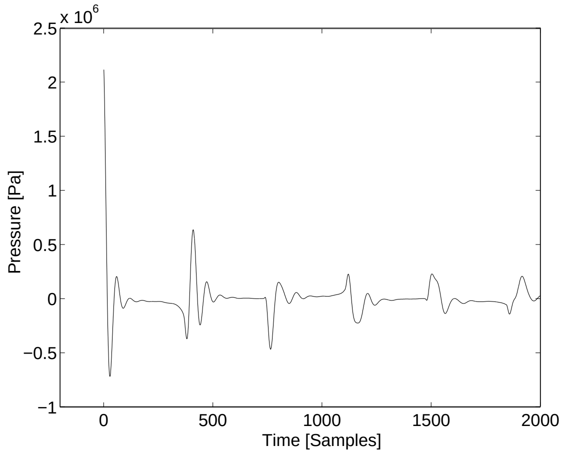

Figure 7: *Impulse response of a brass instrument simulated at a sampling rate of 44100 kHz. The parameters of the model are adjusted to fit the measures of a trumpet.*

next step the lip model was connected to the instrument. Simulations yield results similar to those published in [7], where a similar lip model and a measured impedance function of a real instrument was used.

7. CONCLUSIONS

In this paper the way from a continuous physical model to a sound synthesis algorithm is shown for a brass instrument. Basic physical models for air column, mouthpiece and lips are introduced. Then the continuous models are turned into the discrete domain. The air column model is solved by performing the FTM procedures, the mouthpiece model and the lip model are solved by performing Laplace-transformation, impulse-invariant-transformation and inverse z-transformation. The discrete models are connected successfully by eliminating a delay-free loop. Feasibility of FTM based sound synthesis for brass instruments is demonstrated by implementing the algorithm in MATLAB. The simulation results show conformity to the basic properties of real brass instruments but also the need for improvement in order to achieve a more realistic sound. On one hand it is possible to use a more sophisticated lip model, on the other hand the instrument model itself can be modified. Two suggestions seem to be useful to achieve a more exact instrument model. Modeling frequency-dependent damping close to the characteristic of real brass instruments may be possible when extending equation (6) with an additional damping term $d_3\dot{\Phi}''$. Using a high-pass impedance as radiation load at the point x_{b2} may also yield improved results.

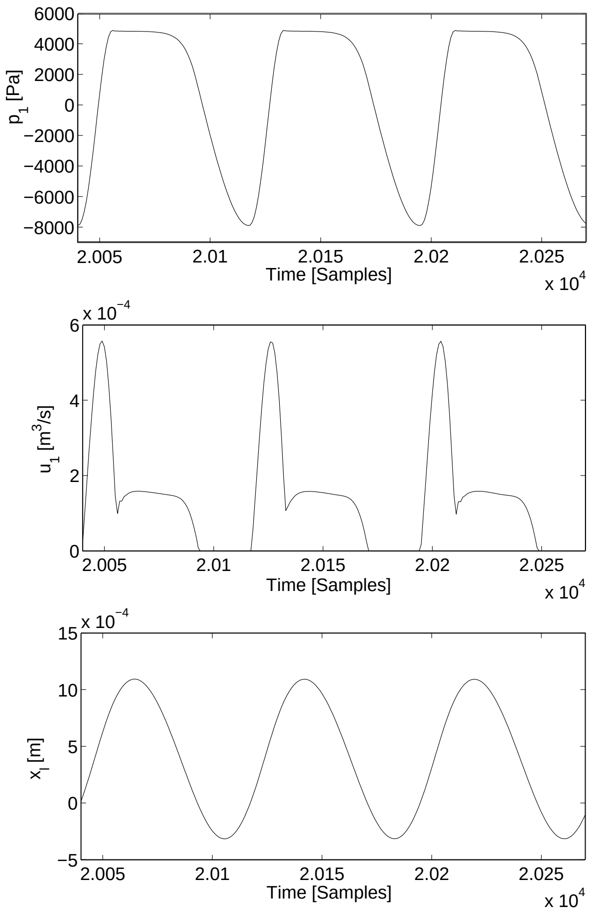

Figure 8: *State variables p_1, u_1 and x_1 of a trumpet tone. Simulation was carried out at a sampling rate of 44100 kHz.*

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