MEASURING DIFFUSION IN A 2D DIGITAL WAVEGUIDE MESH

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ABSTRACT

The digital waveguide mesh is a method by which the propagation of sound waves in an acoustic system can be simulated. An important consideration in modelling such systems is the accurate modelling of reflection characteristics at boundaries and surfaces. A significant property of an acoustic boundary is its diffusivity. In fact partially diffuse sound reflections are observed at most real acoustic surfaces and so this is an important consideration when implementing a digital waveguide mesh model. This paper presents a method for modelling diffusion that offers a high degree of control. The model is implemented with varying amounts of diffusivity, and a method for measuring its diffusive properties is outlined. Results for the model are presented and a method to calculate the *diffusion coefficient* is described.

1. INTRODUCTION

The digital waveguide mesh is a technique used to model the propagation of sound waves in 2-D and 3-D acoustic systems [1, 2] and can therefore be used in musical instrument and room acoustic modelling. It is a model that, by its nature, incorporates the effects of diffraction and wave interference [2, 3]. This is an advantage when it is compared to other room acoustic modelling techniques such as the image source method [4] and ray-tracing [5].

A *specular* reflection occurs at a smooth surface when the angle of the reflected sound wave is equal to the angle of incidence. A *diffuse* reflection occurs when a sound wave reflects from a rough boundary and results in a redistribution of the sound energy across a range of angles. In the most extreme case, the energy is spread evenly in every direction, whatever the angle of incidence [6].

The behaviour of the propagating sound waves in an acoustic system can be affected significantly by the diffusive characteristics of the boundaries. Accurate modelling of this effect in the digital waveguide mesh is therefore required. Previous work details the successful implementation of a highly diffusive boundary in a 2-D mesh using a quadratic residue diffuser [7]. However this method limits the amount of control over the diffusivity of the surface and also causes complications if other boundary characteristics are to be modelled, such as frequency dependent absorption.

Another technique has been developed that simulates diffusion by randomly rotating the incident waves as they approach the boundary of the mesh [8]. This model allows the diffusivity of the boundary to be controlled and is lossless, however it introduces an error.

In this paper, a new method in which this error is eliminated is used, as presented in a previous work [9]. A random probability density function, used to control the random angles by which the incident waves are rotated, is also applied to the model and investigated.

In order to fully test the diffusive properties of these models, and to compare them with other acoustically diffusive boundaries, an accurate method for measuring the diffusivity is required. In this paper we apply current methods in measuring diffusion to our model. An approach is outlined that can be used to measure the *diffusion coefficient*, which is an attempt to quantify diffusivity proposed by RPG Diffusor Systems [10].

2. THE DIGITAL WAVEGUIDE MESH

The digital waveguide mesh is an extension of the 1-D digital waveguide used for physical modelling synthesis [11]. It is made up of discrete time bi-directional delay lines connected by scattering junctions, or nodes, which are arranged to form 2-D or 3-D structures. The scattering junctions act as spatial and temporal sampling points.

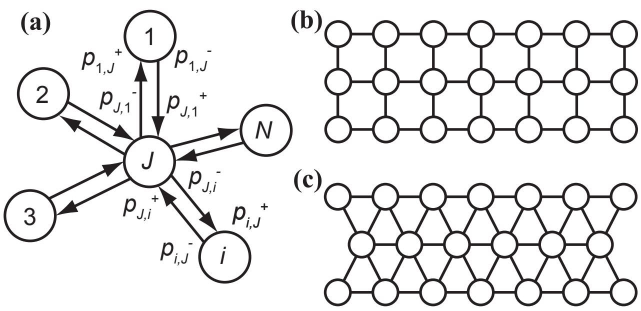

Figure 1: *(a) A general scattering junction J with N connected waveguides for i = 1,2,. . . N; (b) a 2-D rectilinear mesh structure; (c) a 2-D triangular structure.*

Figure 1(a) shows a scattering junction, J, with N neighbours, i = 1, 2, . . . N. These are connected together by delay lines, or waveguides. The sound pressure in a waveguide is represented by p_i, which is defined by the sum of the two signals p_i^+ and p_i^-:

$$p_i = p_i^+ + p_i^- \tag{1}$$

These travel in opposing directions along the bidirectional delay line. When two junctions are considered, i and J, the signal $p_{i,J}^+$ represents the incoming signal to junction i along the waveguide connected to junction J. Similarly, the signal $p_{i,J}^-$ represents the outgoing signal from junction i.

By connecting scattering junctions together, it is possible to model wave propagation in 2-D and 3-D spaces. Different mesh topologies can be used to model the same physical structure. For instance a 2-D space can be modelled using either a rectilinear mesh or a triangular mesh, diagrams of which can be seen in Figures 1(b) and 1(c) respectively. The choice of topology dictates the number of neighbours that each scattering junction has.

The sound pressure at a lossless scattering junction, P_J, can be found using Equation 2, where $p^+_{J,i}$ represents the incoming pressure signal at a connecting waveguide and Z_i represents its impedance. Again, N is the number of connecting waveguides at the junction.

$$P_J = \frac{2\sum_{i=1}^{N}\frac{p^+_{J,i}}{Z_i}}{\sum_{i=1}^{N}\frac{1}{Z_i}} \tag{2}$$

The scattering junctions are separated by bi-directional unit delay lines. This means that the input to a scattering junction is equal to the output from a neighbouring junction into the connecting waveguide at the previous sampling time step:

$$p^+_{J,i} = z^{-1}p^-_{i,J} \tag{3}$$

Equations 1, 2 and 3 are collectively termed the scattering equations of the system.

A limitation of the digital waveguide mesh is dispersion error. This results in an inconsistency in the velocity of wave propagation that is dependent on both its frequency and direction of travel. The latter implies that different mesh topologies will have different dispersion characteristics and this has been investigated previously [12] as well as methods to reduce this error [13]. The triangular mesh used in this work has been shown to offer the best dispersion characteristics in the 2-D plane, such that it becomes almost independent of wave direction and is therefore a function of frequency only.

3. A DIFFUSION MODEL USING CIRCULANT MATRICES

At each time step, incoming signals at the scattering junctions are processed according to the scattering equations and new signals are passed out ready to be received by the neighbouring junctions at the next time step. By multiplying the incoming signals with circulant matrices, it is possible to rotate them around a node by any angle, with the result that the direction of the propagating wave at that point is altered. This may be proven mathematically [8], but it is only exact if the connecting waveguides are uniformly distributed around the junction.

The incoming signals at the nodes adjacent to the boundary can be multiplied with circulant matrices, forming a diffusion layer in such a way that the directions of the incoming waves are randomly altered just before they are reflected.

By applying the rotation to this adjacent layer formed from 6-port air nodes, rather than to the n-port boundary nodes where n may take any value from 1-6 according to the geometry of the space, rotation errors are avoided as the connecting waveguides are always uniformly distributed around the junction.

The result of this transformation effectively simulates diffusion and also ensures that energy is conserved. By varying the range over which the rotation angle is allowed to change, the amount of diffusion can be controlled.

3.1. Implementation in the 2-D triangular waveguide mesh

The incoming signals at a 6-port junction in a 2-D waveguide mesh can be rotated by an angle φ if they are multiplied with a circulant matrix, **A**, whose coefficients can be calculated using the following set of eigenvalues, X:

$$X = \begin{bmatrix} 1 & e^{j\varphi} & e^{j2\varphi} & -1 & e^{-j2\varphi} & e^{-j\varphi} \end{bmatrix} \tag{4}$$

An inverse Fourier transform, performed on these eigenvalues, yields 6 real numbers that sequentially make up the first row of coefficients, x_0 ... x_5, in the circulant matrix, A. The coefficients in subsequent rows can be calculated as follows:

$$A = \begin{bmatrix} x_0 & x_1 & \dots & x_5 \\ x_5 & x_0 & \dots & x_4 \\ \dots & \dots & \dots & \dots \\ x_1 & \dots & x_5 & x_0 \end{bmatrix} \tag{5}$$

The matrix is then multiplied with the incoming signals at the junction, s_i and a new set of rotated signals, s'_i is achieved:

$$A\begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_5 \end{bmatrix} = \begin{bmatrix} s'_0 \\ s'_1 \\ \vdots \\ s'_5 \end{bmatrix} \tag{6}$$

Diffusion is simulated by randomly altering the amount of rotation, φ, of the incoming signals at each of these nodes for each sample step. The amount of diffusion that is modelled can be controlled by limiting the algorithm to a range of angles. As an example, to simulate a relatively smooth wall the maximum random angle can be set to 5 degrees in either direction. More complete diffusion can be achieved by increasing this angle.

As a result of this diffusing layer technique, waves that approach the boundary are usually rotated twice. Once as they approach the boundary and a second time as they travel away from it after being reflected. This can be compensated by halving the randomly chosen rotation angles at the junctions adjacent to the boundary. Undesirable effects may occur, however, when large rotation angles are applied because waves may be rotated more than twice or even just once, depending on the angle of approach and the amount of rotation applied.

3.2. Diffusion Control

To accurately model real diffusive boundaries, the existing model can be enhanced by applying random angles of rotation via an appropriate probability density function. This could be used, for example, to ensure that small angles of rotation are more likely to occur than large angles for a surface of low diffusivity. An approach commonly used in geometric acoustics is to use a probability density function that gives diffusion characteristics obeying *Lambert's cosine law* [14]. In this work a Normal probability distribution function is used.

4. DIFFUSION MEASUREMENT IN A 2-D DIGITAL WAVEGUIDE MESH

Due to the nature of the digital waveguide mesh, it is difficult to predict the behaviour of any boundary model without measuring its effects in a simulation. Measurements can then be analysed, just as digital recordings of sound from the real world can be analysed. In order to accurately measure the effects of the diffusion model presented in this paper, a procedure is proposed and implemented for measuring the diffusion coefficient of the boundary, based on the method outlined in [10].

The diffusion coefficient is a measurement of the degree to which a surface uniformly scatters incident sound. *Directional* diffusion coefficients can be measured for arbitrary angles of incidence. If a sufficient number of directional diffusion coefficients are obtained for different incident angles, they can be averaged to give the *random incidence* diffusion coefficient of the surface. The coefficient can be measured for different frequency ranges, giving information about the frequency dependency of the diffusion model.

4.1. Measurement and Geometry

Ideally the test should take place in a space with no acoustic boundaries, or an anechoic chamber, so that the results are not interfered with by waves reflected at the perimeter surfaces of the room. It is also possible to avoid this problem by making the waveguide mesh sufficiently large in relation to the relative distances between objects in the test.

In the test, a patch of the diffusive surface is placed in the middle of the space, and receivers (microphones) are placed in a semicircle around its face, as illustrated in Figure 2. An impulse is then applied at a source, which is placed at an arbitrary point on another outer semicircle, with a radius larger than that of the receiver semicircle. In the case of a 2-D waveguide mesh, measurements can only be taken on a single plane. However the test could be extended to a 3-D waveguide mesh, either taking measurements on two orthogonal planes or at different points on a hemisphere. Figure 2 is a diagram showing the test geometry employed in order to achieve the results presented in this paper.

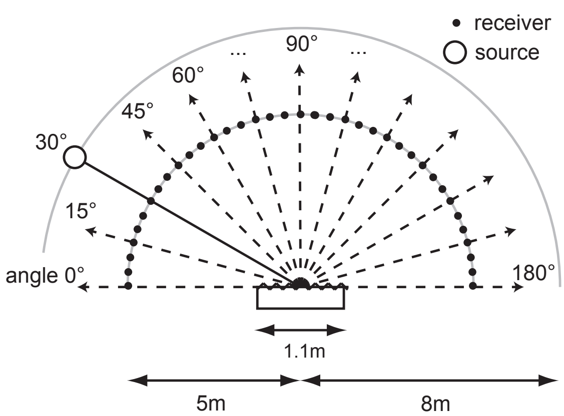

Figure 2: *Diagram showing the setup used for diffusion testing leading to the measurement of the diffusion coefficient.*

Distances between different reference points in the model should not be less than the distances presented here. Ideally, the distances should be greater, so that true *far field* conditions can be achieved [10]. Far field conditions are achieved when spherical sound waves emanating from the point source are allowed to travel far enough that they can be considered plane waves when they interact with the surface under test.

For more accurate measurements, the angular resolution of the receivers should be 5 degrees. Also, it is necessary that at least 75% are outside the path of the specular reflection from the source, or specular zone.

In order to calculate the diffusion coefficient of a certain diffusive object, impulse responses should be obtained for the space both with the diffusor present, $h_1(t)$ and in an empty space (without the diffusor), $h_2(t)$. It is then possible, at each receiver position, to measure the *diffusor impulse response*, or the impulse response that results only from the signal that has reflected from the diffusive surface, $h_3(t)$:

$$h_3(t) = h_1(t) - h_2(t) \tag{7}$$

If the tests are performed in the real world using a loudspeaker and microphone, it is suggested that the impulse response of the source/microphone pair is also measured, so that it can be taken into account in later calculations using a process of deconvolution [10]. However this is not necessary in digital waveguide mesh simulations, as signals can be directly injected and measured.

The diffusor impulse response is calculated at each receiver position, and the frequency analysis of these results yields information about the diffusive qualities of the surface for a given angle of incidence.

4.2. The Diffusion Coefficient

For a fixed source position, the directional diffusion coefficient can be measured in each 1/3 octave band using the amplitude levels of the diffusor impulse response signals at the corresponding frequencies, L_i, measured at each of the n receivers. The auto correlation of these measurements gives the diffusion coefficient, d:

$$d = \frac{(\sum_{i=1}^{n} E_i)^2 - \sum_{i=1}^{n} E_i^2}{(n-1)\sum_{i=1}^{n} E_i^2} \quad ; \quad E_i = 10^{L_i/10} \tag{8}$$

4.3. Case Study

A 2D triangular waveguide mesh is implemented (with mesh sample rate F_s = 44100Hz), and in the first of three tests, the diffusive effects of a flat, specular surface of width 1.1m is measured. In the second and third tests, a diffuse surface is implemented using the technique outlined in Section 3. To determine the random angles of rotation, a Normal probability function is used such that the mean angle of rotation is 0^o and the standard deviation is 23^o in the second test and 45^o in the third test. For later reference, these three tests are labelled *SD00*, *SD23* and *SD45* in order of increasing levels of diffusivity.

In all three simulations, a low pass filtered impulse is applied at a source, placed at an angle of 30^o to the tangent of the surface under test and at a distance of 8 m, as shown in Figure 2. An angular resolution of 5^o is used for the receivers, with the total number

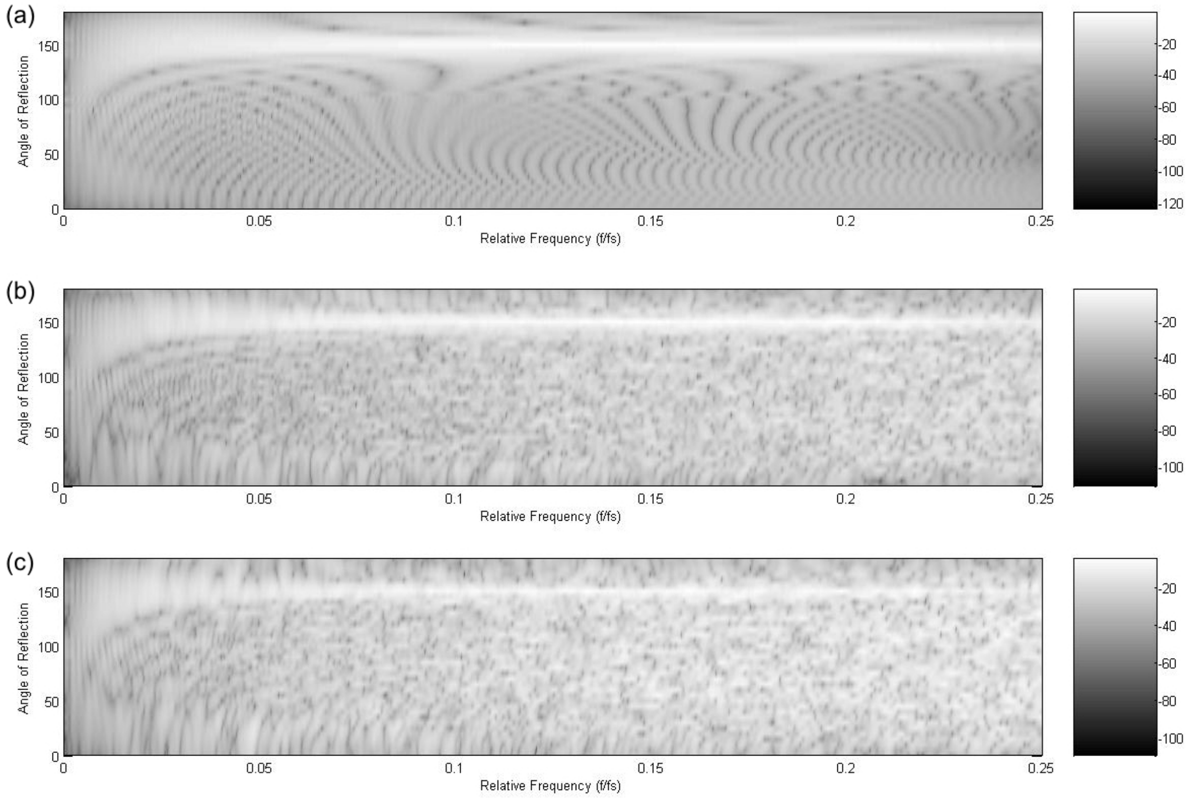

Figure 3: *Spectrograms showing reflection magnitude varying with angle of reflection across the semicircular range of receivers for (a) a specular surface (SD00), (b) a diffuse surface (SD23) and (c) a second diffuse surface (SD45).*

of receivers therefore being 37. The simulation is then run for sufficient time to allow the propagating signal to travel to the surface under test and then to subsequently reflect and propagate to the receivers, which are placed in a semicircle at a distance of 5m from the surface at angles running from 0^o through to 180^o with respect to the surface. In order to obtain the diffusor impulse response at each receiver position, an impulse response is measured in an empty room, so that the direct responses from source to receiver can be removed from the response (7). The room is sufficiently big that reflections from its perimeter do not interfere with the results.

4.4. Results and Discussion

Frequency analysis of the resulting diffusor impulse responses is presented in Figure 3 in the form of spectrograms. For each angle the impulse response is zero-padded and a 4096-point FFT is applied. The results are presented using an x-axis relative frequency scale up to a quarter of the sampling rate as typically a digital waveguide mesh simulation is limited to giving valid results within this bandwidth only [13].

In the first test, patterns of constructive and destructive interference are observed, caused by phase differences in the reflected waves. However in SD23 and SD45 this uniform behaviour is eliminated and the spectrum becomes much more random and noisy, as would be expected with reflections from an actual diffusing surface.

As the amount of diffusion is increased by increasing the standard deviation in the diffusing layer normal distribution function, the energy observed at the angle of specular reflection (150^o) reduces, and the total energy recorded across all other angles increases.

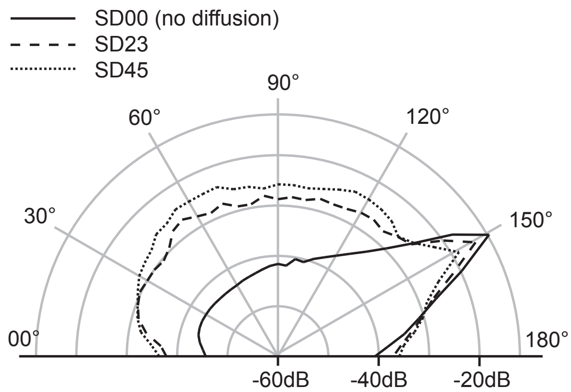

Figure 4: *Polar response showing scattering of sound energy after reflection for each test*

Note also that reflection deteriorates in all three cases in the low frequency region due to the wavelength of the incoming sound wave being larger than the width of the diffusing object. The object in question is 1.1m wide which implies that an incident sound wave will not be reflected as effectively below about 310Hz. This gives a relative frequency value of 0.007 which is in good agreement with the results as presented, where it can be seen that the sharpness of the specular reflection gets progressively worse below 0.025.

The observations are confirmed by the polar plot shown in Figure 4. In this graph, the RMS levels of the diffusor impulse responses are plotted against the angle of reflection.

From these results it is possible to calculate the diffusion coefficients for the three different boundary models. In order to give the diffusion coefficient across a range of frequencies, the RMS levels in each 1/3 octave band of the frequency responses should be used. The diffusion coefficients across all frequencies can also be calculated using the RMS levels of the impulse responses. This approach yields the following diffusion coefficients, d_n, for the three boundaries modelled in the tests:

SD00: d_1 = 0.0336

SD23: d_2 = 0.1539

SD45: d_3 = 0.5606

The diffusion coefficients confirm that the increase in the standard deviation in the random probability function results in an increase in the diffusivity of the modelled surface.

5. CONCLUSIONS

From the results it can be seen that the model for diffusion presented in this paper can be used to model variable diffuse reflections at the boundary of a 2-D triangular waveguide mesh with a high degree of control. The method for measuring the diffusivity of boundary models, described in this paper, can be used to accurately measure the diffusive properties of any surface modelled in the digital waveguide mesh paradigm, for any given angle of incidence. The results are valid at least for frequencies above 5% of the sampling frequency, however it may be possible to obtain results for lower frequencies by increasing the relative distances between the source/receivers and the surface under test and by also increasing the size of the diffusive surface.

The diffusivity measured using this technique can then be quantified by calculating the diffusion coefficient, meaning that they can be conveniently compared with other acoustically diffusive surfaces. The diffusion coefficient representation holds more information regarding the diffusivity of a surface than the *scattering coefficient*, used by geometrical room acoustic modelling systems, which defines the fraction of the scattered energy that is diffused [10].

Future research will focus on simulating frequency dependent diffusive surfaces, extending the diffusion model to 3-D digital waveguide meshes and further methods to measure diffusivity.

6. ACKNOWLEDGEMENTS

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