THE MODIFIED CHAMBERLIN AND ZÖLZER FILTER STRUCTURES

Duane K. Wise

Consultant
Boulder, Colorado, USA
dwise@wholegrain-ds.com

ABSTRACT

The Chamberlin and Zölzer filter structures run a risk, albeit small, of requiring a large value for one of their tuning coefficients, which may lead to performance issues. A simple modification leads to alternative structures that place an absolute bound on that coefficient while retaining the signal flow topology. The modified structures also affect the pole distribution. In the case of the Chamberlin structure, the changes upon modification can be interpreted as favorable.

1. BACKGROUND

1.1. The Chamberlin and Zölzer Filter Structures

Chamberlin, in [1], outlines a versatile filter structure based on embedded digital integrators. Zölzer, in [2], attributes this structure in a slightly modified form to a prior publication by N. G. Kingsbury. Also in [2], Zölzer makes an intuitive leap from the Chamberlin and Kingsbury works that results in a filter structure that bears his name.

Figure 1 presents the signal flow graph of the Chamberlin filter structure. The Kingsbury structure is virtually identical save that the input node is at a different location. In this paper, the filter structure will herein be called the Chamberlin structure.

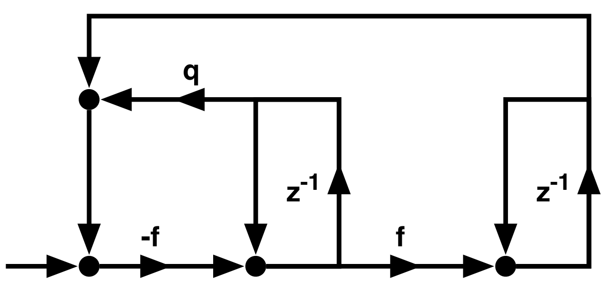

Figure 1: *The Chamberlin filter structure.*

The transfer function that the Chamberlin structure generates is

$$H_C(z) = \frac{N(z)}{1-(2-f^2-f\cdot q)\cdot z^{-1}+(1-f\cdot q)\,.z^{-2}} \quad (1)$$

where $N(z)$ is a linear combination of node-specific transfer function numerators within the filter structure, which is beyond the scope of this paper.

Figure 2 presents the signal flow graph of the Zölzer filter structure. The structure depicted in the figure is slightly different from that presented in Zölzer's publication but results in an identical transfer function denominator.

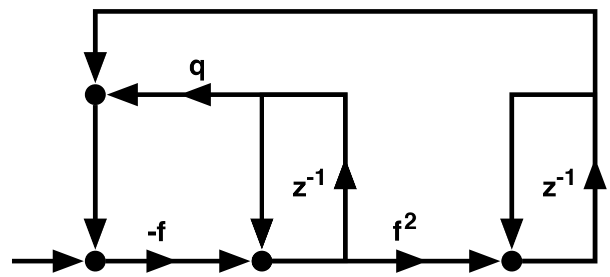

Figure 2: *The Zölzer filter structure.*

The transfer function that the Zölzer structure generates is

$$H_Z(z) = \frac{N(z)}{1-(2-f^3-f\cdot q)\cdot z^{-1}+(1-f\cdot q)\,.z^{-2}} \quad (2)$$

where $N(z)$ is a linear combination of nodes as in (1).

1.2. Advantages

The Chamberlin filter structure can conveniently render several useful transfer functions simultaneously, such as highpass, lowpass, bandpass, and allpole [1], a trait that is also true of the Zölzer structure. In addition, the tuning coefficient f maps approximately to the tuning frequency parameter and the coefficient q maps approximately to the reciprocal of the Q parameter of the resulting poles.

Both filter structures offer a favorable distribution of poles when their coefficients are quantized. Figure 3 shows the pole distribution of the Chamberlin filter when f and q are quantized in linear steps. Figure 4 shows the pole distribution of the Zölzer filter with equivalent coefficient quantization. The pole distribution plots are more amenable to tuning with logarithmically scaled frequency and Q parameters [2] [3].

2. LIMITATIONS

In order to tune these filter structures to any stable transfer function, a formula to determine valid tuning ranges, based on reflection coefficient calculation, is given in [4]. Applying this formula to the Chamberlin structure, the following relations for f and q must apply for stability.

$$0 < f < 2 \quad \wedge \quad 0 < q < \frac{4-f^2}{2f} \quad (3)$$

The relation (3) implies that, as coefficient f approaches zero, the value of coefficient q can become quite large, with no absolute

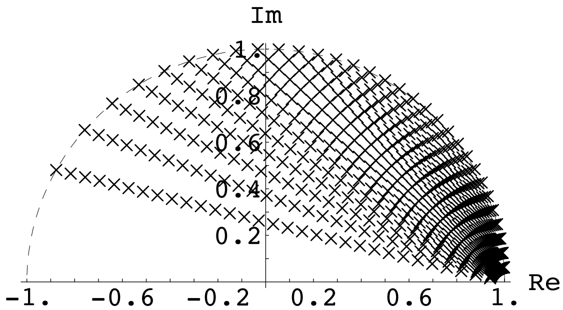

Figure 3: *Linearly quantized pole distribution of the Chamberlin structure.*

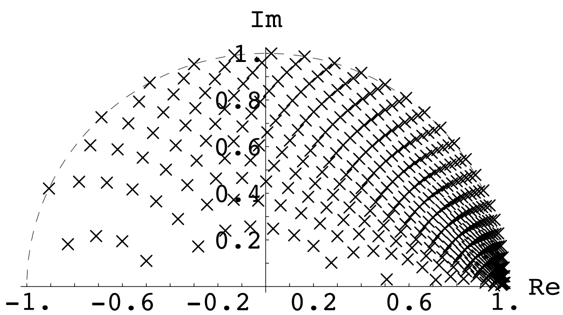

Figure 4: *Linearly quantized pole distribution of the Zölzer structure.*

bound. To illustrate, Figure 5 shows the value of q for a 10-octave tuning range ending at Nyquist $\times 20.48$ kHz $/24$ kHz and a range of Q from $1/32$ to 32.

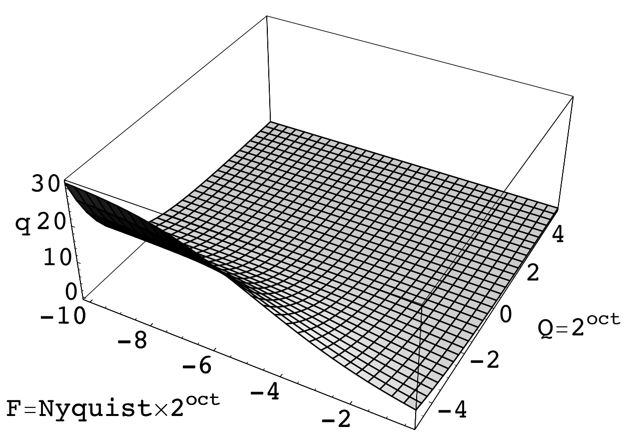

Figure 5: *Value of coefficient q for the Chamberlin structure.*

An inspection of Figure 3 and Figure 4 above shows that the Chamberlin structure has significantly lower pole density near Nyquist (-1 on the complex plane), whereas the Zölzer structure is more able to fill that void.

The Zölzer structure has the following relations for f and q to insure stability.

$$0 < f < 2^{2/3} \quad \wedge \quad 0 < q < \frac{4 - f^3}{2f} \tag{4}$$

Similar to (3), the relation (4) implies that the upper range of q is essentially boundless. However, Figure 6 shows that the Zölzer structure is less susceptible to q growth over the same tuning range.

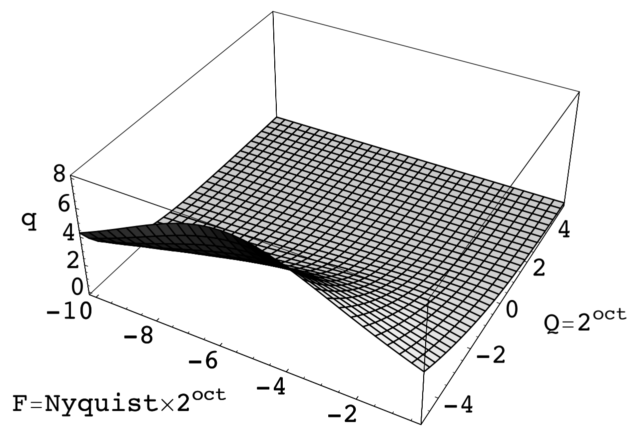

Figure 6: *Value of coefficient q for the Zölzer structure.*

While the wide range of the q parameter is clearly problematic for a fixed point implementation of these filter structures, a high level of q raises the signal level at the input node relative to the other summation nodes, which can raise the noise level at the input node regardless of whether fixed point or floating point math is utilized. Note that quantization noise generated at the summation nodes feeding the delays will eventually be multiplied by the q coefficient in the recursive network.

3. MODIFICATION

A simple modification to the above filter structures can help alleviate the above limitations. As illustrated in Figure 7, the $-f$ multiplier can be moved against the signal flow within the recursive loop to yield a $-f$ multiplier from the rightmost delay and a $-fq$ multiplier from the other delay. Figure 7 depicts what is herein called the Modified Chamberlin filter structure.

The transfer function that the Modified Chamberlin structure generates is

$$H_{MC}(z) = \frac{N(z)}{1 - (2 - f^2 - fq) \cdot z^{-1} + (1 - fq) \cdot z^{-2}} \tag{5}$$

where $N(z)$ is a linear combination of node-specific transfer function numerators within the filter structure. Applying the modification depicted in Figure 7 to the Zölzer filter structure results in what will be called the Modified Zölzer filter structure. The transfer function that it generates is

$$H_{MZ}(z) = \frac{N(z)}{1 - (2 - f^3 - fq) \cdot z^{-1} + (1 - fq) \cdot z^{-2}} \tag{6}$$

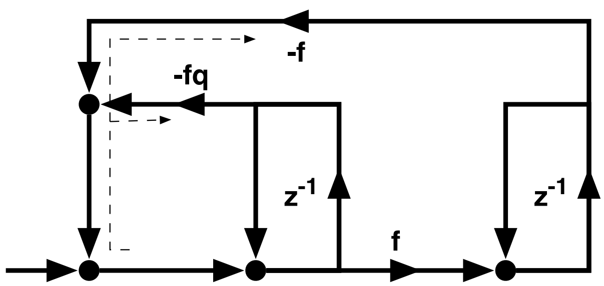

Figure 7: *Multiplier modifications resulting in the Modified Chamberlin filter structure.*

3.1. The Modified Chamberlin Filter Structure

The modified filter is still tuned with two coefficients, which are called f and fq in this paper, but the role of fq as opposed to q is slightly different. To illustrate this difference, Figure 8 shows the pole distribution of the Modified Chamberlin structure using the same quantization step size as that for Figure 3. Comparing the two figures indicates that the Modified Chamberlin structure retains the advantage of higher pole density towards DC while compensating for the Chamberlin structure's low pole density at Nyquist.

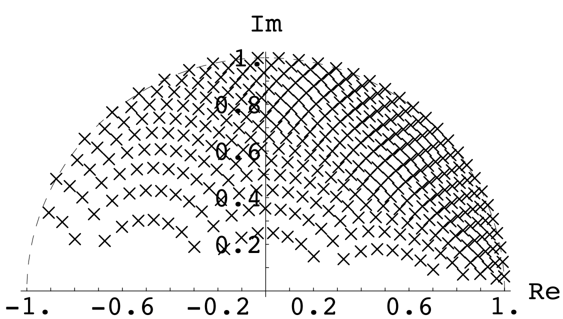

Figure 8: *Linearly quantized pole distribution of the Modified Chamberlin structure.*

The following relation must apply for the Modified Chamberlin structure to render a stable transfer function.

$$0 < f < 2 \quad \wedge \quad 0 < fq < \frac{4 - f^2}{2} \tag{7}$$

Plugging in the valid range of f into the range of fq shows that the value of fq cannot exceed 2 (also note that the z^{-2} terms of the modified transfer function denominators (5) and (6) cannot have their absolute values exceed unity in order for the denominator to remain in the stability triangle). Figure 9 shows the value of fq for the same tuning parameters as in Figure 5 and Figure 6. As compared to the Chamberlin structure in Figure 5, the range of the fq coefficient is considerably smaller than that of q. This aids in fixed point coefficient storage (note that both f and fq can fit in the range of a traditional unsigned fixed point data word) and in lowering the relative level at the input node, which can improve the overall noise performance.

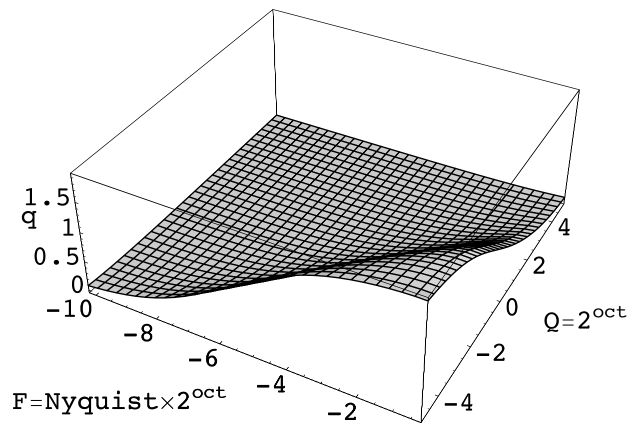

Figure 9: *Value of coefficient fq for the Modified Chamberlin structure.*

3.2. The Modified Zölzer Filter Structure

Applying the modification to the Zölzer filter structure yields less dramatic changes in performance. Figure 10 shows the pole distribution of the Modified Zölzer structure using the same quantization step size as that for Figure 4 and Figure 8. A comparison of Figure 10 with Figure 4 indicates that the two distributions are similar with the Modified Zölzer distribution having lower overall density than the Zölzer structure. Comparing Figure 10 with Figure 8 can make a case that the Modified Chamberlin structure outperforms the Modified Zölzer structure in pole distribution under coefficient quantization, which cannot be made when comparing the nonmodified versions of the two structures.

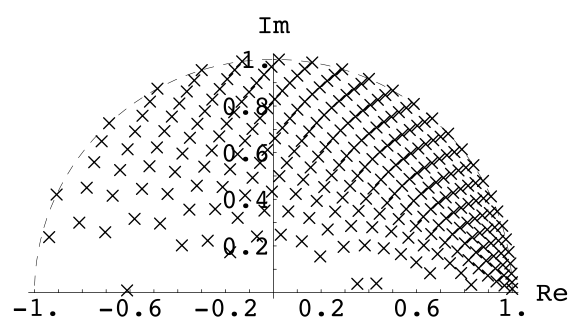

Figure 10: *Linearly quantized pole distribution of the Modified Zölzer structure.*

The following relation must apply for the Modified Zölzer structure to render a stable transfer function.

$$0 < f < 2^{2/3} \quad \wedge \quad 0 < fq < \frac{4 - f^3}{2} \tag{8}$$

The value of the fq tuning coefficient for the Modified Zölzer structure is identical to the fq coefficient for the Modified Chamberlin structure, given in Figure 9 (note that the z^{-2} terms of the modified transfer function denominators (5) and (6) are equivalent and depend solely on fq). As the nominal range of q in the Zölzer structure is of lower magnitude than that of its Chamberlin counterpart, the noise improvement of the Modified Zölzer structure over the Zölzer structure is not as significant.

4. CONCLUSIONS

This paper offers for consideration a simple modification to the Chamberlin and Zölzer filter structures. This modification is offered not as a replacement to the two structures but as an alternative design that may aid in issues that involve a large value of the coefficient q. The q coefficient in the established structures has no absolute bound, though in most tuning circumstances the coefficient value is not large in magnitude. In the case where the value of q is large, performance issues may arise regardless of whether fixed point or floating point math is employed in the filter implementation.

The Modified Chamberlin and Modified Zölzer filter structures replace the coefficient q with a coefficient called fq, which represents the product of f and q in the established structures. The absolute upper bound of fq is 2 in all stable tuning configurations. The pole distribution of the modified structures is different from that of their canonical counterparts. The pole distribution of the Modified Chamberlin structure can compare favorably to that of the Chamberlin structure. It is unlikely that the pole distribution of the Modified Zölzer structure can be considered preferable. The modified filters do not affect the signal flow topology of the structures, which retains most, if not all, of the advantages of the established structures outlined in [1] and [2].

5. ACKNOWLEDGEMENTS

The author wishes to thank Mic Chorn for his assistance in the preparation of this paper, and to the anonymous reviewers for their invaluable time, attention, and feedback. The filter analysis is performed with Mathematica version 5.2 (Wolfram Research, Inc.; 2005).

6. REFERENCES

[1] H. Chamberlin, *Musical Applications of Microprocessors, Second Edition*. Hayden Books, 1985.

[2] U. Zölzer, *Digital Audio Signal Processing*. Chichester, UK: J. Wiley & Sons, 1997.

[3] D. Wise, "A survey of biquad filter structures for application to digital parametric equalization," 1998, *Presented at the 105th AES convention*, preprint #4820.

[4] J. Markel and A. Gray Jr., *Linear Prediction of Speech*. New York: Springer-Verlag Berlin Heidelberg, 1976.