THE INFLUENCE OF SMALL VARIATIONS IN A SIMPLIFIED GUITAR AMPLIFIER MODEL

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ABSTRACT

A strongly simplified guitar amplifier model, consisting of four stages, is presented. The exponential sweep technique is used to measure the frequency dependent harmonic spectra. The influence of small variations of the system parameters on the harmonic components is analyzed. The differences of the spectra are explained and visualized.

1. INTRODUCTION

Most guitar players love to experiment with different sounds and use varying equipment like amplifiers or effect units. Thanks to *amp modeling*, the signal processor based simulation of famous guitar amplifiers, it has become easier for today's musicians to have a high number of sounds combined in one device. The common way to do amp modeling is to disassemble a real amplifier circuit, analyze the separated blocks accurately, and write a software that simulates the transfer behavior step-by-step. In doing so, a copy of the original device with similar sound properties can be created, but quality and similarity of the result depend on the complexity and the analysis' accuracy. Hence, good simulations are still computationally intensive and require high-performance equipment.

In this paper guitar amplifiers are understood as complex systems with highly nonlinear behavior. By measuring the frequency dependent harmonic spectrum, a unique "finger print" is recorded, identifying the system's behavior for a given sinusoidal input. The influence of small variations of the system parameters to the output spectrum can be characterized by this measurement technique. This study can help to analyze the influence of filters and transfer curves to the sound. Even though a strongly simplified model is used, the tendencies are in evidence. In addition to the recording of these measurable parameters, a listening test is presented to statistically identify the changes in timbre for the same model. The assignment between measurable values and timbre attributes like "aggressive" or "warm" is desired.

2. GUITAR AMPLIFIERS

For the amplification of electric guitars, special amplifier and loudspeaker combinations are common, which clearly differ in their transfer behavior from normal Hi-Fi units. In use are *combo* amplifiers combining amplifier and speaker(s) in one enclosure or *stacks*, consisting of amplifier (*top*) and box separately. The amplifiers show a strong nonlinear performance and enforce a volitional distortion of the guitar signal. The tone of the undistorted instrument is rarely of interest, the popular guitar sound is always associated with a more or less strong deformation of the original signal. Due to their excellent sound behavior when driven in such circuits, tubes are still ruling todays guitar amplifiers. The reproduction is done via special loudspeaker cabinets, where the used speakers make a main contribution to the sound because of the non flat frequency response showing many resonances. The main stages of a common valve amplifier are [1]:

- The *input stage* with triode valves responsible for the preamplification of the guitar signal.
- The *tone stack* suppressing the DC component from the signal and providing simple equalization.
- The *phase splitter* providing both original and phase inverted signal as required for the symmetrical feeding of the following power amplifier.
- The *power amp stage* with pentode valves processing individual amplification of the original and the phase inverted signal.
- The *output transformer* executing the subtraction of both signals delivered by the power amp stage to achieve the doubling of the signal amplitude. The transformer is also necessary for the impedance matching to the connected speaker (typically 8 or 16 Ω).
- One or more *speakers* mounted in an open or closed cabinet. Commonly used speakers range from 10 to 12 inches.

Each of these stages comes with non-ideal transfer characteristics. When combining all stages to the overall system the analysis gets very complex. The nonlinearity of a typical phase splitter circuit is explained in [2]. The analysis and synthesis of the parasitic nonlinear behavior of a guitar loudspeaker cabinet can be found in [3].

3. MEASUREMENT OF NONLINEAR SYSTEMS

The nonlinearity of a system leads to additional spectral content in the output signal. This means that the output spectrum will show new parts at discrete frequencies kf_1 when it is caused with a sine wave of frequency f_1, e.g. for a sine wave of $1\,\mathrm{kHz}$, the first overtone occurs at $2\,\mathrm{kHz}$ and the second at $3\,\mathrm{kHz}$. In music, these integer multiples are called *overtones*. In physics it is common to use the term *harmonic* with a different index relating to the order: the excitation frequency is called *fundamental*, the first overtone k_2, the second overtone k_3 and so on.

A common gage to describe nonlinear systems is the specification of the *total harmonic distortion* (THD) caused by the system, defined as the ratio of the sum of the powers of all harmonic components H_i to the power of all harmonic components H_i plus the fundamental H_1 [4]:

$$\text{THD} = \frac{\sqrt{H_2^2 + H_3^2 + ... + H_N^2}}{\sqrt{H_1^2 + H_2^2 + H_3^2 + ... + H_N^2}}. \quad (1)$$

Depending on the systems assembly, the value of the components in the output spectrum can vary extremely both over frequency and input amplitude.

3.1. Exponential Sweep Technique

The sine sweep technique is a well-known tool to measure impulse responses (e.g. of rooms) featuring a high resolution and robustness against time variance. A useful variant of this method, introduced by Farina [5], uses a real valued sweep

$$x(t) = \sin\left(\frac{\omega_1 T}{\ln(\frac{\omega_2}{\omega_1})}\left(e^{\frac{t}{T}\ln(\frac{\omega_2}{\omega_1})} - 1\right)\right) \quad (2)$$

whose frequency increases exponentially from the start frequency ω_1 to the end frequency ω_2, with sweep length T. In addition, an inverse sweep $x^{-1}(t)$ has to be computed, fulfilling the condition

$$x(t) \otimes x^{-1}(t) \approx \delta(t - T). \quad (3)$$

When $x(t)$ is applied to a system, the convolution of the measured output signal $y(t)$ with $x^{-1}(t)$ leads to the impulse response

$$h(t) = y(t) \otimes x^{-1}(t). \quad (4)$$

For linear systems, $h(t \leq T)$ is zero and the linear impulse response is given by $h(t > T)$. When the measured system is afflicted with memoryless nonlinearity, $h(t)$ will show additional spikes, which are located prior to the impulse response of the linear part of the system. For an exponentially swept sine, these spikes denote separated impulse responses, which belong to the harmonic components and are defined as *harmonic impulse responses* (HIR) [6]. Beginning with the 2nd order component k_2 to the left of the main pulse the HIRs are separated neatly in time one after another. The position of the spike leading to the i_{th} order HIR can be computed from

$$t_{\text{HIR}(i)} = t_{\text{FUND}} - \frac{\log_2(i)}{\text{sweep rate}}, \quad (5)$$

where i is the order of the harmonic component and t_{FUND} is the arrival time of the fundamental response. This relation is illustrated in Figure 1. For high orders, the distance between the spikes is getting smaller and the responses will overlap. A remedy is to record a longer sweep and/or to use a higher sampling rate. By executing a FFT on each harmonic response, the frequency dependent harmonic distortion is computed. Figure 2 shows the frequency response for the fundamental wave and the first harmonics k_2 and k_3. In this plot the harmonics are displaced to the left so that the spectral content is plotted under the corresponding excitation frequency. The components H_i, as used in (1), can be read out easily from the plot. The advantage of this technique is, that even heavily distorted systems can be measured accurately. The linear response and the harmonic responses up to high orders are separated in time leading to segmented spectra for all orders.

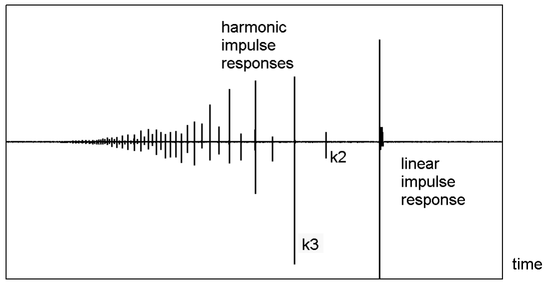


Figure 1: *Measurement of a strongly nonlinear system, with the linear impulse response and the harmonic responses.*

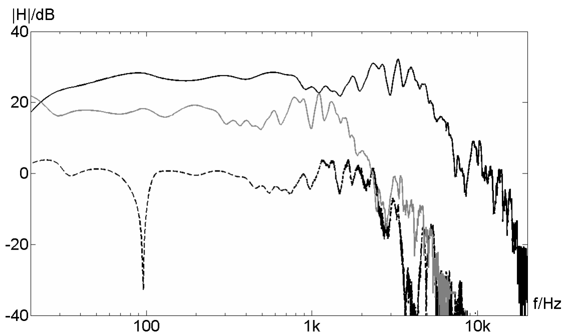


Figure 2: *The harmonic components k_2 (dashed) and k_3 (gray) plotted under the corresponding fundamental (black).*

4. SYSTEM ANALYSIS

4.1. Simplified Model of a Guitar Amplifier

To analyze the behavior of guitar amplifiers a software model had to be developed offering the possibility of easy modifications. Disregarding the linear or nonlinear effects of transformer and phase splitter, the described circuit can be simplified to a four stage model as shown in Figure 3. It consists of the following four stages:

1. a characteristic curve with static nonlinear behavior as found in preamplifiers,
2. a linear filter representing the tone stack,
3. a second characteristic curve as found in the power amplifier and
4. the loudspeaker simulation performed by a FIR filter with the impulse response of a measured guitar cabinet.

This model was implemented on a professional recording PC using the software *Cubase 4*. The characteristic curves were processed

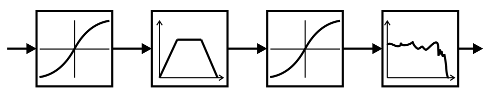


Figure 3: *The four stage amplifier model.*

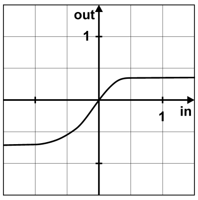


Figure 4: *The nonlinear transfer curve of the preamp.*

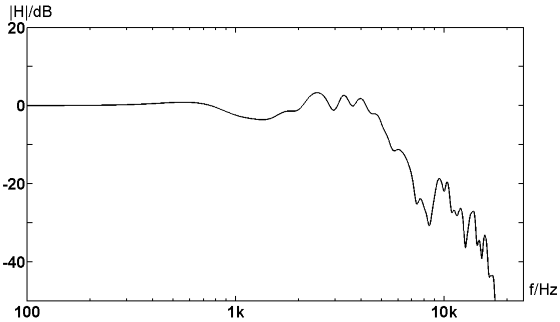


Figure 5: *Frequency response of the Blue Bulldog loudspeaker.*

with the free plug-in *Func Shaper*[1], providing the transfer curves by means of mathematical expressions. The filter sections were integrated with the parametric EQ of the Cubase plug-in Q. For the speaker simulation the freeware plug-in *keFIR*[2] was used. The signal processing was done with help of the *Powercore Firewire* from TC ELECTRONIC and the *Fireface 400* Interface from RME.

4.2. Reference Setup

The reference setup was designed as a strongly simplified amplifier model of the *normal* channel of a classic VOX *AC30* guitar amplifier. The transfer curve for the stages I and III are shown in Figure 4. In this case the same soft clipping transfer curve is used both for pre- and power amplifier. The second stage performs the low cut filter caused by a coupling capacitor plus the attenuation of the high frequencies caused by the phase splitter. The frequency response of the loudspeaker simulation was taken from measurements with the original *Blue Bulldog* speaker and is shown in Figure 5. All parameters are defined in Table 1. For stages I and III the functions are given as pseudo code according to the syntax of the used plug-in.

4.3. Modified Setups

To analyze the presented four stage amplifier model, some modifications were done in all stages. In the following 21 modified setups only one stage was changed in the parameters while all other stages remain as given in the reference setup.
Stage I: The characteristic curve of a 12AX7 triode was changed both to a hard clipped and soft clipped curve (setup 1, 2). The effect of a symmetric / asymmetric transfer curve was tested as well (setup 14, 15).
Stage II: A peak filter with mid frequency $f_m = 800\,\mathrm{Hz}$ and two octaves bandwidth was added to the model. The cases boost (setup 5) and cut (setup 6) with gain = $20\,\mathrm{dB}$ were applied. In setup 17 the cut-off frequency of the low cut filter was changed.
Stage III: Similar to stage I, the transfer curve was modified both to a hard clipped and soft clipped curve (setup 3, 4) and to symmetric / asymmetric behavior (setup 20, 21).
Stage IV: To simulate the influence of changes in the loudspeaker response, a peak filter was added to the original speaker. The parameters are: $f_m = 800\,\mathrm{Hz}$, bandwidth Q = 2 octaves, and gain = $\pm 6\,\mathrm{dB}$ (setup 7 and 8).
In addition, further modifications in all stages were done including some curves with extreme or "crazy" characteristics. An overview over all setups is given in Table 2. Figures of all important curves are shown the appendix.

[1] Func Shaper Version v0.5, http://www.rs-met.com/
[2] keFIR, http://habib.webhost.pl/

5. MEASUREMENT OF ALL SETUPS

The reference setup and all modifications were analyzed with the described measuring technique. To ensure a sufficient resolution at high orders, very long sweeps of 30 s were used at a sampling frequency of 44.1 kHz. For each setup five recordings were done capturing the important range of the input amplitude. This range was found to be $-60\,\mathrm{dBV}$ to $-20\,\mathrm{dBV}$. With 1 reference and 21 modifications the number of sweep measurements is 22×5 at all, without counting additional ones used for ensuring accuracy. From each measurement the linear response and all harmonic responses up to the order 9 were computed. The following nomenclature is used to identify the singular responses

$$H_i^{j,V}(k) \qquad (6)$$

with

- the harmonic order i beginning with 1 for the fundamental,
- the index of the measured setup $j = \{1, \ldots, 21, \mathrm{R}\}$ and
- the used input amplitude $V = \{-60\,\mathrm{dBV}, \ldots, -20\,\mathrm{dBV}\}$ in steps of $10\,\mathrm{dBV}$.

5.1. Interpretation and Comparison

For the comparison all measurements were arranged in a meaningful way one-to-one, leading to a total of 15 pairs. That way symmetric and asymmetric or hard- and soft clipped transfer curves were faced. The arrangements are shown on the left part of Table 3.

Table 1: *The parameters of the reference model.*

No.	Parameters
I	SoftClip$(0.5(x+1.41)^2 - 1, a, b, c, d)$ with a=-0.753, b=0.351, c=0.875 and d=0.625; gain=31.16 dB and volume=5.89 dB.
II	low-cut filter with f_c=56 Hz and a high shelving filter with f_c=1973 Hz, gain -2.5 dB.
III	SoftClip$(0.5(x+1.41)^2 - 1, a, b, c, d)$ with a=-0.639, b=0.681, c=0.667 and d=0.771; gain=17.68 dB and volume=-11.10 dB.
VI	A low-cut filter with cut-off frequency f_c=20 Hz and a 512 tap long FIR filter

Table 2: *Overview over the settings for the modified setups.*

No.	Stage	Description	Parameters
1	I	hard clipped preamp curve	c=0, d=0
2	I	soft clipped preamp curve	c=1, d=1
3	III	hard clipped power amp curve	c=0, d=0
4	III	soft clipped power amp curve	c=1, d=1
5	II	added peak filter boost	f_m=800 Hz, Q=2 octaves, gain=20 dB
6	II	added notch filter cut	f_m=800 Hz, Q=2 octaves, attenuation=20 dB
7	IV	peak filter added to loudspeaker (boost)	f_m=800 Hz, Q=2 octaves, gain=6 dB
8	IV	notch filter added to loudspeaker (cut)	f_m=800 Hz, Q=2 octaves, attenuation=6 dB
9	I	unrealistic *freeform* curve	z=x+a sin(2 π b x);e=tanh(c z) with a=0.56, b=2.28, c=1.84, drive=20.2 dB and volume=0 dB
10	I	rectifier *half-abs*	abs(x)/2 +x/2 with drive=31.16 dB and volume=0 dB
11	I	rectifier *abs*	abs(x) with drive=31.16 dB and volume=0 dB
12	I	*squared* curve	$x^2/2 + x/2$ with drive=31.16 dB and volume=0 dB
13	I	*tilted* curve	e=below(-a,x); f=above(b,x); if(e,x,-1-2x)+if(f,x,1-2x); with a=0.33, b=0.33, drive=26.11 dB and volume=0 dB
14	I	symmetric soft	softClip(select(x, c tanh(a x), d tanh(a x))); with a=3.125, c=1, d=1, drive=31.16 dB and volume=5.89 dB
15	I	asymmetric soft	same as No.14, but with d=0.5, drive=31.16 dB and volume=5.89 dB
16	IV	added high-shelving filter	f_s=1428 Hz, gain=8 dB
17	II	low-cut frequency moved	f_c=140 Hz
18	I	reduced gain	drive=8 dB
19	III	reduced gain	drive=0 dB
20	III	symmetric soft	softClip(select(x, c tanh(a x), d tanh(a x))); with a=3.125, c=1, d=1, drive=17.68 dB and volume=-11.10 dB
21	III	asymmetric soft	same as No. 20, but with d=0.5, drive=17.68 dB and volume=-11.10 dB

As a metric for the similarity the averaged difference between two transfer curves $H_i^{j1}(k)$ and $H_i^{j2}(k)$ is introduced, computed by

$$\Delta_i^{(j1,j2),V} = \frac{1}{N}\sum_{k=0}^{N}\left|H_i^{j1}(k) - H_i^{j2}(k)\right| \tag{7}$$

with the mean over all frequency bins k. A small value for Δ denotes a high similarity between two curves. Prior to the computation of this value, it makes sense to prepare the measurements. First a smoothing has to be applied to the frequency responses, as explained in [7], to ensure insensitivity against additional noise. With 1/3 octave smoothing good results were achieved. The second step is to align the magnitude plots in a meaningful way. Therefor the fundamental responses H_1 were normalized to a mean value of 0 dB and the harmonic responses were aligned accordingly. The similarity is then calculated for the measurements done with the input levels V=-30 dBV through -50 dBV. Table 3 gives an overview over the similarity between all measured curves. Specified is the maximum value calculated by

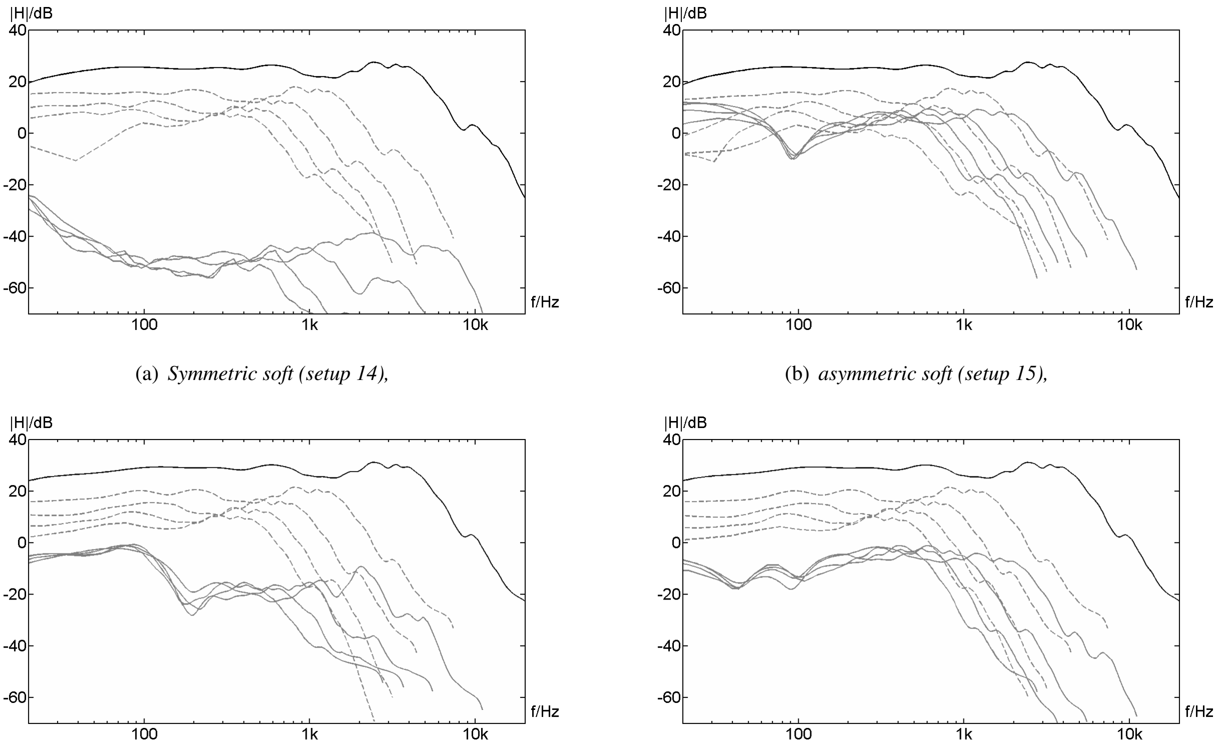
$$\max\left(\Delta_i^{(j1,j2),V}\right), V = \{-50\,\text{dBV}, \ldots, -30\,\text{dBV}\}, \tag{8}$$

for all pairs j_1, j_2 and the harmonics' index i. The mapping is ++ for very high similarity and - - for no observable similarity. Although this representation is just a rough estimation, some patterns are visible.

Because of the high number of pairs only a selection of the results can be presented in this paper. Some observations reflect the well-known behavior of nonlinear transfer curves, e.g. that symmetrical nonlinearities (setup 14, 20) will cause odd order harmonics while asymmetrical nonlinearities cause even order distortion products. Further on it can be seen that the characteristic of the fundamental (row f_1) consists mainly of the loudspeakers' response. Since this LTI system has its position at the end of the chain, all harmonics show more or less the same form.

5.2. Selected Results

Asymmetric vs. symmetric soft transfer curves: A symmetric transfer curve driven in the preamplifier, as simulated in setup 14, causes high levels for all odd harmonics, but minor levels for the even harmonics. Partially is the difference bigger than 40 dB, see Figure 6(a). The third harmonic is the strongest component at all input levels.
A different trend can be observed for the asymmetric preamp curve, as depicted in Figure 6(b). Again, all odd and all even harmonics run in groups parallel, but the difference is much smaller. For small input amplitudes the second harmonic can be stronger than the dominating third component. No difference can be found for the fundamental response. The same tendency can be found comparing the measurements from the second transfer curve, setup 20 and 21. This condition can be seen clearly in Table 3, too.
Soft clipped vs. hard clipped transfer curves: The differences in the measurements of the hard clipped preamp curve (setup 1) and the soft clipped one (setup 2) are not as distinctive. Again, the



(a) *Symmetric soft (setup 14),* (b) *asymmetric soft (setup 15),*

(c) *hard clipped (setup 1)* (d) *and soft clipped (setup 2).*

Figure 6: *Fundamental (black), odd (dashed gray) and even order harmonic responses (solid gray), measured with input level* $-40\,\mathrm{dBV}$.

Table 3: *The similarity* Δ *(max) for all arrangements, as introduced in equation (8).*

pair		harmonic component i								
j_1	j_2	f_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
1	2	++	○	++	○	+	○	+	○	+
5	6	○	○	-	○	- -	○	- -	-	- -
9	R	+	-	○	- -	○	- -	○	- -	-
12	R	++	- -	-	○	-	-	- -	-	- -
18	R	++	-	- -	-	- -	- -	- -	- -	- -
3	4	++	+	++	+	+	+	+	+	○
7	8	+	+	+	+	+	+	+	+	+
10	R	++	- -	○	-	-	○	-	○	-
13	R	++	- -	○	- -	-	- -	- -	- -	-
19	R	++	-	- -	-	- -	- -	- -	- -	- -
14	15	++	- -	++	- -	+	- -	○	- -	○
16	R	++	○	+	+	+	+	++	+	+
11	R	++	- -	-	-	-	○	- -	-	- -
17	R	++	○	++	+	++	+	++	+	+
20	21	++	-	++	-	+	-	++	- -	++

changes in the odd order components are smaller than in the even harmonics. The main deviation is given for the input amplitude $-40\,\mathrm{dBV}$, where the hard clipped system shows a dip for all even harmonics, contrary to the soft clipped system, see Figures 6(c) and 6(d).

5.3. Listening Test

Modifications in the setup will of course lead to changes in the timbre of the perceived sound. Some changes will result in a complete different sound, others may be almost inaudible. For the analysis which modifications result in an audible change, and how those changes can be described, an online listening test has been developed. Three short sound clips of an electric guitar played with different technique were processed with the different setups. The same arrangement of setups as presented in Table 3 is compared and the user has to rate different attributes describing the sounds (e.g. "warm", "aggressive" or "transparent"). The listening test can be found on the website

`http://www.amptest.de`

The purpose of this is to join and compare the results of the measurements and the listening test and find relations between perceived sound and measurable parameters.

6. DISCUSSION

The harmonic spectrum of a guitar amplifier is wide. For the comparison made in section 5, only the harmonic components up to k_9 have been taken into account. Disregarding higher harmonics of course leads to an inaccuracy. But the idea is to bring out the tendencies of the changes, for this reason the limitation to only 9 harmonics is sufficient. The reference model sounds good, but due to the simplifications it works static. This means, the model can not react to the dynamic of the playing, like real amplifiers and (extensive) simulations will do. But it was not the task to develop an excellent model and the cognition of this study holds for real amplifiers models as well.

For the comparison of asymmetric and symmetric preamplifier curves, it has to be considered that the output level of the asymmetric curve is lower. Therefor the following second transfer curve is not driven in the same amplitude range. But the same tendency of the harmonics can be found in setup 20 and 21, where the second transfer curve is modified in the same way.

7. CONCLUSIONS

In this paper, the influence of small variations in a guitar amplifier model is analyzed. A strongly simplified model is presented, consisting of two nonlinear transfer functions, a linear filter in between, and a loudspeaker simulation. By using the exponential sweep technique, the frequency dependent harmonic spectra of all systems are measured, giving an identification of the system's behavior for a given input amplitude. In addition, the system parameters are varied and the influence to the output spectra are characterized and compared. It is shown, that the spectral components can vary in a wide range, depending on the performed modifications. The differences are explained and visualized for selected results.

8. REFERENCES

[1] U. Zölzer, Ed., *DAFX: Digital Audio Effects*, John Wiley & Sons, Inc., New York, NY, USA, 2002.

[2] S. Möller, M. Gromowski, and U. Zölzer, "A measurement technique for highly nonlinear transfer functions," in *Proc. Digital Audio Effects (DAFx-02)*, Hamburg, Germany, Sept. 26-28 2002.

[3] D. Yeh, B. Bank, and M. Karjalainen, "Nonlinear modeling of a guitar loudspeaker cabinet," in *Proc. Digital Audio Effects (DAFx-08)*, Espoo, Finland, Sept. 1-4 2008.

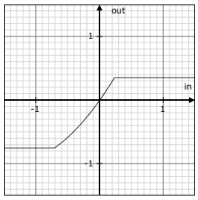
[4] S. Temme, "Audio distortion measurements," Tech. Rep., Brüel & Kjaer, 1992.

[5] A. Farina, "Simultaneous measurement of impulse response and distortion with a swept-sine technique," in *108th AES Convention*, Paris, France, Feb. 19-24 2000.

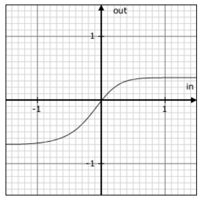
[6] S. Müller, "Measuring transfer-functions and impulse responses," in *Signal Processing in Acoustics*. 2008, vol. 1, pp. 74–76, D. Havelock, S. Kuwano and M. Vorländer (Eds.), Springer.

[7] P. Hatziantoniou and J. Mourjopoulos, "Generalized fractional-octave smoothing of audio and acoustic responses," *J. Audio Eng. Soc.*, vol. 48, no. 4, pp. 259–280, April 2000.

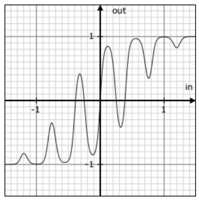
A. APPENDIX: CHARACTERISTIC CURVES OF THE MODIFIED SETUPS



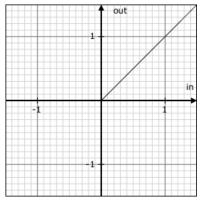
(a) Hard clipped,



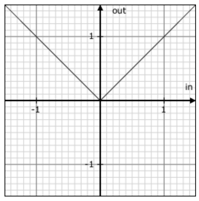
(b) soft clipped,



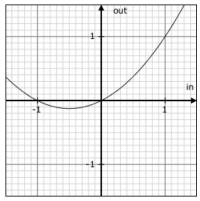
(c) freeform,



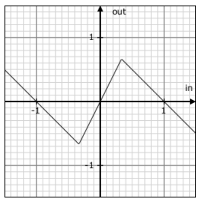
(d) half-abs,



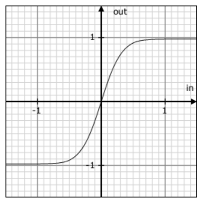
(e) abs,



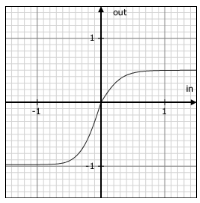
(f) squared,



(g) tilted,



(h) symmetric and



(i) asymmetric soft.

Figure 7: The modifications of the transfer curves.