RENDERING OF AN ACOUSTIC BEAM THROUGH AN ARRAY OF LOUDSPEAKERS

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ABSTRACT

This paper addresses the problem of rendering a virtual source through loudspeaker arrays. The orientation of the virtual source and its aperture determine its radial beampattern. The methodology we present here imposes that the wavefield in a predetermined listening area best approximates the desired wavefield in the least squares sense. With respect to the traditional techniques the number of constraints is much higher than the number of loudspeakers. As a consequence, the loudspeaker coefficient vector is the solution of an over-determined equation system. Moreover this system may be ill-conditioned. In order to solve these issues, we resort to a least squares inversion combined with a *Singular Value Decomposition* (SVD) to attenuate the problem of ill-conditioning. Some experimental results show the feasibility and the issues of this methodology.

1. INTRODUCTION

In this paper we will investigate on the use of an arbitrarily shaped loudspeaker array to approximate the soundfield generated by a source placed at a location behind the array. The source is characterized by an arbitrary radial pattern and its location and orientation in space can be chosen at random. Our ability to accurately control the parameters of an acoustic beam has a twofold use. First, we envision to exploit this technique to provide a tool that is able to excite the environment with a signal that exhibits a pre-determined structure both in time and space. We intend to infer the geometric and acoustic properties of the environment from the acquired response. Moreover, we intend to use the acoustic beamshaping engine to generate the elementary components that are used for the rendering of soundfields.

We recall that an arbitrary wavefield can always be decomposed into a superposition of elementary waves. Ambisonics approximates the soundfield as the decomposition of spherical functions ([1],[2]). The wavefield synthesis (WFS) ([3],[4],[5]) is based on the Huygens principle that states that every wavefront can be decomposed into a superposition of elementary spherical wavefronts emitted from secondary sources. Each loudspeaker, therefore, is independently controlled in order to operate as a secondary source. In our approach we represent the soundfield as a superposition of beams originating from multiple image sources. We observe that a technique that is able to approximate a desired wavefield is useful for both the above purposes of controlled excitement of the environment and of the rendering of an arbitrary soundfield.

The goal of all the techniques for soundfield rendering with loudspeaker arrays is to compute weights (or filters when more than one frequency is considered) to be applied to each loudspeaker signal to obtain the desired shape of wavefronts.

In [6] the authors control the direction of the maximum on the beampattern of the loudspeaker array by computing an array factor that imposes a suitable delay and gain to each loudspeaker. However, the methodology in [6] is not able to accurately control the shape of the beampattern but just the direction of the beam. Moreover, the authors focus on the farfield case.

Generalized Sidelobe Cancelling (GSC) [7], originally presented to steer the sensitivity of microphone arrays, allows to put multiple constraints on the beampattern. However, we can place a number of constraints on the beampattern that is limited by the number of loudspeakers, thus preventing an effective control of the shape of the beampattern.

An interesting solution for the shaping of an arbitrary beampattern can be found in [8]. Here the authors propose to perform the shaping into two steps: in a first stage the design is taken back to the the farfield. The nearfield beampattern is then obtained from the farfield one. The validity of the beampattern in the broadband makes this algorithm very interesting. However, for some position of the virtual source the design of the beampattern is difficult to obtain.

In this paper we propose an alternative technique to simulate by means of an array of M loudspeakers the arbitrary beampattern of a virtual source. More specifically, we define N test points in a listening area of arbitrary shape. We impose that the wavefield on the test points best approximates the wavefield produced by the virtual source with the specified beampattern. This condition yields a system of N equations whose unknowns are the M loudspeaker weights. In order to achieve a smooth beampattern, the number of test points is much higher than the number of loudspeakers. Therefore, the system of equations is overdetermined. When the virtual source is in near field, the solution of the system is generally ill-conditioned [9]: we resort to a SVD to attenuate this problem.

The rest of the paper is structured as follows: Section 2 illustrates the problem and gives an overview of the background. Section 3 describes the proposed solution. Section 4 provides some experimental results to show the feasibility of the proposed approach. Finally, Section 5 draws some conclusions and illustrates some future developments of the framework such as the broadband extension and the rendering of multiple virtual sources.

* The SCENIC project acknowledges the financial support of the Future and Emerging Technologies (FET) programme within the Seventh Framework Programme for Research of the European Commission, under FET-Open grant number: 226007

2. PROBLEM STATEMENT AND BACKGROUND

In this Section we will formulate the problem of beamshaping and we will give an overview on the existing techniques both in near and far fields. Consider the problem of rendering a narrowband signal in a pre-determined area of interest, which we shall call *listening area* throughout the rest of the paper. As mentioned in the introduction, many works in the literature have addressed this problem but mainly for the far field case. In particular, several methodologies can be brought to the "delay-and-sum" beamshaper. A vector of complex coefficients applied to the loudspeakers enables to selectively leave undistorted the signal towards a desired direction, while attenuating the others. Consider a uniform linear array of $m = 1, \dots, M$ loudspeakers placed in points $\mathbf{p}_1, \dots, \mathbf{p}_M$. The geometry of the problem in the far field case is depicted in Figure 1. Consider the origin of the reference frame in the first loudspeaker, so that the phase displacements will be referred to this emitter. The term d is the distance between adjacent loudspeakers. We will now assume valid the far-field hypothesis, i.e. the wavefront that impinges on the listening area is planar. The sound pressure $p(t)$ at a test-point in the listening area for a waveplane propagating towards the direction θ is described by the equation[1]:

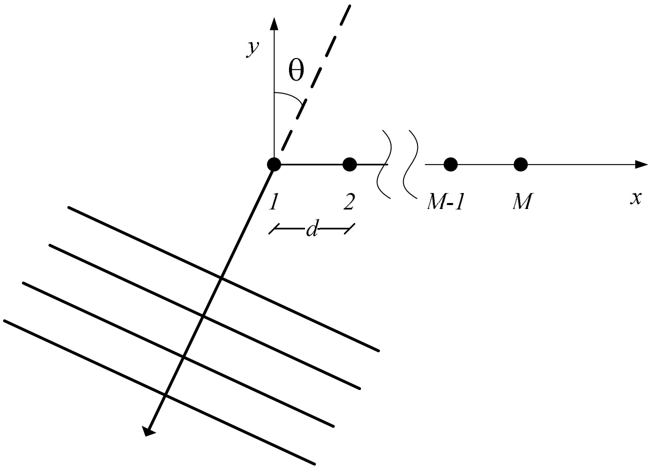

Figure 1: Geometry of the far field "delay-and-sum" beamshaping technique.

$$p(t) = \mathbf{f}^H \mathbf{g}(\theta) s(t) ,$$

where $s(t)$ is the source signal; $\mathbf{f}$ is the vector of unknown complex coefficients applied to the loudspeakers; and $\mathbf{g}(\theta)$ is the propagation vector from each loudspeaker to the listening point for the emitting direction θ:

$$\mathbf{g}(\theta) = [1 \quad e^{-j\omega \frac{d \sin(\theta)}{c}} \quad \dots \quad e^{-j(M-1)\omega \frac{d\sin(\theta)}{c}}]^T , \qquad (1)$$

where c is the sound speed and ω is the central frequency of the narrow-band signal $s(t)$. We omit the dependency of the propagation vector from ω since we assume it as a constant. The term $\Psi = \mathbf{f}^H \mathbf{g}(\theta)$ is the spatial filtering-function of the array for a waveplane of direction θ [10]. More specifically, the vector $\mathbf{f}$ has the following form:

$$\mathbf{f} = [H_1 \quad H_2 \quad \dots \quad H_M]^H , \qquad (2)$$

where H_m is the coefficient applied to the m-th loudspeaker. In order to leave undistorted the signal emitted towards direction θ while minimizing the total energy emitted, the following constrained minimization rule is used [10]:

$$\hat{\mathbf{f}} = \arg \min_{\mathbf{f}} \mathbf{f}^H \mathbf{g}\mathbf{g}^H \mathbf{f} \qquad \text{subject to } \mathbf{f}^H \mathbf{g}(\theta) = 1 . \qquad (3)$$

The solution of the above problem through the method of Lagrange multipliers leads to:

$$\hat{\mathbf{f}} = \mathbf{g}(\theta)/M . \qquad (4)$$

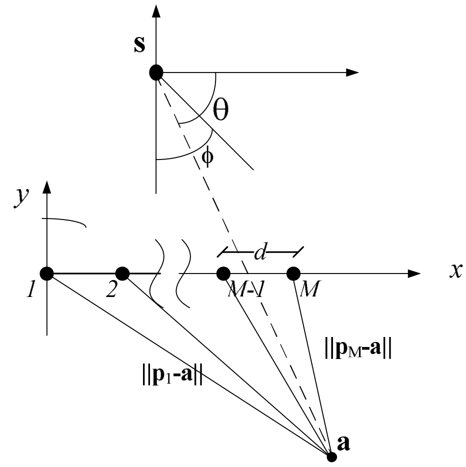

Figure 2: Geometry of the near-field beamshaping system based on a single constraint.

We recall, however, that our goal is to design a beamshaper in the nearfield. If we re-examine the geometry of the problem applied to the nearfield case (see Figure 2), we observe that the solution in eq.(1) in not suitable for our case for the following reasons:

1. as depicted in Figure 2 the distances from each loudspeaker to the listening point are not equal but expressly depend on the position of the test-point and of the loudspeakers;
2. the aperture ϕ is not explicitly controlled in eq.(1). In the literature, this fact is known as the sweet-spot problem.

In order to remove the first limit, we could, theoretically, reformulate the vector $\mathbf{g}$ to account for a variable distance between $\mathbf{p}_1, \dots, \mathbf{p}_M$ and the listening point $\mathbf{a}$. This way, the m-th element in the propagation vector $\mathbf{g}$ is the Green's function from $\mathbf{p}_m$ to $\mathbf{a}$ [11]:

$$g(\mathbf{p}_m, \mathbf{a}) = \frac{1}{4\pi||\mathbf{p}_m - \mathbf{a}||} e^{-j\omega \frac{||\mathbf{p}_m - \mathbf{a}||}{c}} . \qquad (5)$$

According to the new definition, the solution of the constrained minimization is:

$$\hat{\mathbf{f}} = \frac{\mathbf{S}^{-1}\overline{\mathbf{g}}}{\overline{\mathbf{g}}^H \mathbf{S}^{-1} \overline{\mathbf{g}}} ,$$

where $\mathbf{S}$ is a matrix obtained from the Green's functions from each loudspeaker to the test point and $\overline{\mathbf{g}}$ is the Green's function

[1] The symbol H denotes the Hermitian transposition; later in the paper the symbol T will also be used to indicate the simple transposition operation.

from the virtual source position $\mathbf{s}$ to $\mathbf{a}$. Unfortunately, the matrix $\mathbf{S}$ is generally ill-conditioned when working in the near field.

Even though the beamshaper in eq.(6) addresses the problem of the far field case, it does not enable us to accurately control the shape of the beampattern. Generalized Sideobe Cancelling allows the imposition of multiple constraints at the same time on the emitting directions. However, it does not solve the problem of the ill-conditioning in the inversion.

In order to deal with the above problems, we will resort to a different technique. First of all, in order to remove the far-field hypothesis, we impose the constraints on listening points rather than on emitting directions; we than solve the minimization using the SVD. An interesting feature that our solution presents with respect to the state of the art is that we can place the emitters in arbitrary positions, needing only to comply with the spatial Nyquist criterion.

3. PROPOSED SOLUTION

As shown in the previous Section, we have to use different design criteria with respect to the state of the art in order to achieve the desired beampattern: instead of minimizing the energy of the beamshaper, we are interested in controlling the shape of the beampattern.

We consider that the emitters are located at points $\mathbf{p}_1, \ldots, \mathbf{p}_M$. The array is configured according to an arbitrary geometry. The listening area is defined by the points $\mathbf{a}_n, \quad n = 1, \ldots, N$. Figure 3 shows the geometry of the system we will use throughout the rest of the paper. The contribution of the m-th loudspeaker to the

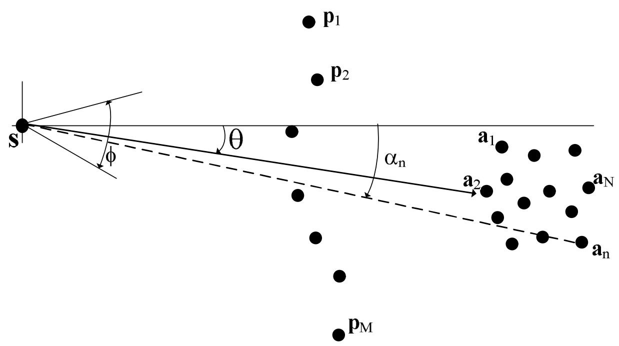

Figure 3: Geometry of the proposed method for near-field beamshaping.

soundfield at $\mathbf{a}_n$ is

$$\Psi_{m,n} = H_m g(\mathbf{p}_m, \mathbf{a}_n) ,$$

where H_m is the coefficient applied to the signal emitted from the m-th loudspeaker, as defined in eq.(2). The soundfield at $\mathbf{a}_n$ is the sum of the signals from all the loudspeakers:

$$\Psi_n = \sum_{m=1}^{M} \Psi_{m,n} = \sum_{m=1}^{M} H_m g(\mathbf{p}_m, \mathbf{a}_n) . \tag{6}$$

The term Ψ_n assumes the role of the spatial response of the loudspeaker array as defined in Section 2. However, the main difference is that the spatial response does not depend solely on the emitting direction, but on the listening point $\mathbf{a}_n$. As stated above, our goal is to render the acoustic beam emitted by a virtual source placed in $\mathbf{s}$ that emits towards the direction θ and with an angular aperture ϕ. The desired response at the point $\mathbf{a}_n$ is

$$\overline{\Psi}_n = g(\mathbf{s}, \mathbf{a}_n)\Theta(\theta, \phi, \alpha_n) ,$$

where $\Theta(\theta, \phi, \alpha_n)$ is the radiation pattern of the virtual source and α_n is the angle under which the n-th listening point is seen from $\mathbf{s}$, as depicted in Figure 3. In Section 4 we will use a Gaussian beampattern. However we remark that this is only a design choice that does not prevent us from using a custom function. Our goal is to approximate the wavefield of the virtual source at the listening points $\mathbf{a}_n, n = 1, \ldots, N$ imposing that the spatial response of the array approximates the spatial response of the virtual source, i.e.: $\Psi_n = \overline{\Psi}_n$. In particular, if we operate on a point-wise basis in the listening area, we impose that

$$\mathbf{g}_n^T \mathbf{h} = g(\mathbf{s}, \mathbf{a}_n)\Theta(\theta, \phi, \alpha_n) , \tag{7}$$

where: $\mathbf{h} = [H_1 \quad H_2 \quad \ldots \quad H_M]^T$ is the vector of unknown coefficients and $\mathbf{g}_n = [g(\mathbf{p}_1, \mathbf{a}_n) \quad g(\mathbf{p}_2, \mathbf{a}_n) \quad \ldots \quad g(\mathbf{p}_M, \mathbf{a}_n)]^T$ is the juxtaposition of the Green's functions from the m-th emitter to the considered listening point.

If we consider all the listening points at once, we obtain the following matrix-formulation:

$$\mathbf{G}\mathbf{h} = \mathbf{r_d} , \tag{8}$$

where $\mathbf{r_d} = [g(\mathbf{s}, \mathbf{a}_1)\Theta(\theta, \phi, \alpha_1), \ldots, g(\mathbf{s}, \mathbf{a}_N)\Theta(\theta, \phi, \alpha_N)]^T$ is the desired response; and $\mathbf{G} = [\mathbf{g}_1, \ldots, \mathbf{g}_N]^T$ is the $N \times M$ propagation matrix from each loudspeaker to each test point. We observe that, in order to obtain a smooth beampattern we need to use $N \gg M$. For the inversion of the system in eq.(8) therefore we have to use least squares-like techniques, in order to obtain $\mathbf{h}$. We will investigate on this in the next paragraph.

3.1. Formulation and solution as an inverse problem

The methodology we present in this paragraph is related to the inverse problems theory [12]. The system in eq.(8) is over-determined and it admits no exact solution. However, an estimation $\hat{\mathbf{h}}$ of the vector $\mathbf{h}$ can be calculated by introducing the pseudo-inverse operation on the matrix $\mathbf{G}$:

$$\mathbf{G}^+ = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H .$$

The loudspeakers weight vector is approximated by:

$$\hat{\mathbf{h}} = \mathbf{G}^+\mathbf{r_d} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{r_d} . \tag{9}$$

In general $\mathbf{G}\hat{\mathbf{h}} \neq \mathbf{r_d}$; however $\hat{\mathbf{h}}$ represents the best solution to the problem in the least squares sense.

The matrix $(\mathbf{G}^H\mathbf{G})$ is positive definite and hence invertible, nevertheless its condition number is not guaranteed to be sufficiently small. In order to avoid instability problems a reconditioning of $(\mathbf{G}^H\mathbf{G})$ is needed. We do so through an SVD decomposition:

$$\mathbf{G}^H\mathbf{G} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H , \tag{10}$$

where $\mathbf{U}$ and $\mathbf{V}$ are, respectively, the left and right singular vectors and $\mathbf{\Sigma} = \mathrm{diag}(\sigma_1, \ldots, \sigma_M)$ is the singular value diagonal matrix and $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_M$. In order to perform the reconditioning, we seek for the greatest index k which guarantees that $\sigma_k/\sigma_1 \geq \xi$. We retain the first k columns and rows of matrices $\mathbf{U}$, $\mathbf{V}$ and $\mathbf{\Sigma}$. The approximate inverse matrix is therefore

$$(\mathbf{G}^H\mathbf{G})^{-1} \approx \mathbf{V}_k\mathbf{\Sigma}_k^{-1}\mathbf{U}_k^H . \tag{11}$$

3.2. Implementation details

We have noticed that the condition number of the matrix $\mathbf{G}^H\mathbf{G}$ mainly depends on the number N of listening points. More specifically, the condition number tends to decrease when N grows, ranging from a value in the order of 10^6 for $N = 100$ to a value in the order of 10^3 for $N = 5000$. The choice of $\xi = 0.01$ leads to a reconditioned version of $\mathbf{G}^H\mathbf{G}$ with condition number equal to 100; this guarantees a good approximation in the SVD based inversion for a variety of situations, independently of the number N of listening points.

The SVD inversion of the matrix $\mathbf{G}^H\mathbf{G}$ is a costly operation. However, we observe that a change in either the radial beampattern of the virtual source or its position correspond only to a change in the vector $\mathbf{r}_d$ of the desired response, as the matrix $\mathbf{G}$ is composed by the Green's functions from each loudspeaker to each test point. As a consequence, the SVD inverse of $\mathbf{G}^H\mathbf{G}$ may be easily precomputed once the positions of loudspeakers and test points are known.

The Tikhonov method for the regularization [13] of the matrix $\mathbf{G}^H\mathbf{G}$ has also been taken into account. However, we noticed that the regularization parameter (i.e. the Tikhonov factor) has to be tuned for each experiment in order to obtain results comparable with the SVD approach. This fact led us to choose the SVD inversion method presented above, as no parameter needs to be tuned at each new position of the virtual source.

4. EXPERIMENTAL RESULTS

This Section is divided into two parts. First we make a comparison, in a specific configuration, between the beampatterns obtained with the proposed technique and with other methodologies already available in the literature. In the second part we assess the accuracy of the synthesized beampattern. We make use of these criteria for different configurations of the virtual source and of the beampattern.

4.1. Comparisons with classic beamshaping methods

In this paragraph we show a comparison between the new beamshaping approach presented in Section 2 and the techniques illustrated in Section 3. In order to perform this comparison, we need to define a geometry that is viable for all the beamshaping techniques. In particular, the "delay-and-sum" beamshaper requires a uniform linear array of emitters; hence we consider an array of $M = 10$ emitters spaced $d = 0.34\ m$ apart. The $N = 180$ test points are placed on a circular arc of radius $\rho = 3\ m$ and centered in the source position $\mathbf{s} = (0, 0)$.

Figure 4 shows the angular responses of the different beamshaping methods: the far-field "delay-and-sum" beamshaper, the near-field beamshaper with a single constraint and the proposed pseudo-inverse technique for beam rendering. The design parameters for the beam are: the orientation $\theta = 0°$; the angular aperture $\phi = 30°$; the working frequency $f = 500Hz$. These parameters are used to define the desired beampattern for the pseudo-inverse beamshaping method. The beampattern is described by a Gaussian function. The aperture ϕ is defined as the angular distance of the two points on the mainlobe having an attenuation of $20dB$ with respect to the maximum value of the function.

The beampattern obtained with the proposed technique has an angular aperture of $30°$, as desired. The other techniques do not allow to control the parameter ϕ, as the aperture is a function of M and d.

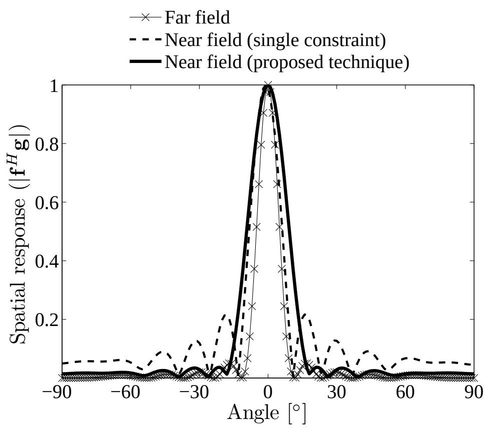

Figure 4: Polar responses of three beamshaping methods (far-field "delay-and-sum", near-field with single constraint and pseudo-inverse method). The beam design parameters are set to $\theta = 0°$ and $\phi = 30°$; the working frequency is $500Hz$.

As far as the sidelobe rejection is concerned (defined as the ratio between the maximum amplitude of the mainlobe and that of the first sidelobe in the beampattern), the solution proposed in this paper attains $28.9dB$ to be compared with $25.9dB$ and $13.3dB$ of the single constraint nearfield and farfield beamshapers, respectively.

4.2. Simulations

In this paragraph we show some simulations to illustrate how accurately the beampattern of the loudspeakers array approximates the desired one. Due to the large amount of simulations conducted, we will show only some synthetic parameters related to the beam obtained. In particular, the accuracy is measured with the following metrics:

- *angular error of the beam*, measured as the absolute value of the difference in the desired and rendered orientation angle;
- *sidelobe rejection*, measured as the ratio (in dB) between the amplitude of the mainlobe and of the first sidelobe of the beampattern;
- *error of the angular aperture of the beampattern*, defined as the difference in the desired and rendered aperture. More specifically, the aperture is defined here as the distance between the two points on the mainlobe where amplitude with respect to the maximum is $-20dB$.

The setup used for the simulations is described in Figure 5:

1. a $1\ m$ radius circular array composed of $M = 32$, equally spaced composed by omnidirectional emitters;
2. a listening area composed of $N = 5000$ points uniformly distributed in a circular region with $0.9\ m$ radius concentric with the array;
3. a virtual source placed at a distance l from the array center, emitting a beam with parameters (θ, ϕ).

In order to satisfy the spatial Nyquist criterion, the frequency of the narrowband signal for all the tests that follow is $f = 300Hz$.

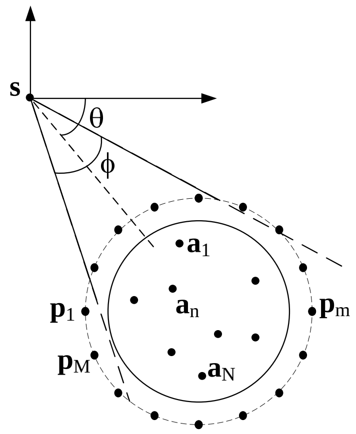

Figure 5: Setup used for the simulations conducted in this Section.

The beampattern is defined as a Gaussian function centered in θ. The angular aperture ϕ of the beam is controlled by tuning the variance of the Gaussian function as described in par.4.1.

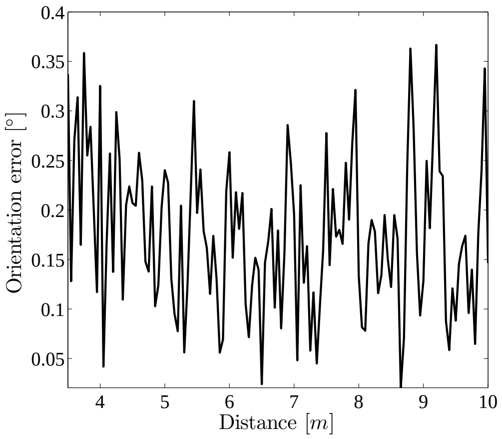

Figure 6: Orientation error for different distances of the virtual sources w.r.t. the array center. The beam parameters are $\theta = 0°$ and $\phi = 10°$

Figure 6 shows the angular error of the rendered beam for different distances of the source, keeping the beam direction and the angular aperture respectively fixed at $\theta = 0°$ and $\phi = 10°$. We observe that the error of the center of the beam is always under $0.4°$. Figure 7 shows the sidelobe rejection for the same configuration (l, θ, ϕ) of Figure 6. The rejection ranges from $5dB$, when the virtual source is located close to the array, to a maximum of about $20dB$ when $l = 10\ m$. The most critical parameter to control is the angular aperture ϕ, as depicted in Figure 8: the beamshaping system suffers from high errors (up to $15°$) when the virtual source is close to the array ($l = 3.5\ m$). However the error rapidly decreases to $5°$ for $l = 6\ m$ and is practically $0°$ when $l = 10\ m$.

We now consider a configuration in which the source is at a distance $l = 2.7\ m$ and the angular aperture is $\phi = 30°$. Figures 9 and 10 show the orientation error and the aperture error, respectively, as a function of θ.

We observe from Figs. 9 and 10 that, as the beam rotates from the central direction, the orientation error and the aperture error in-

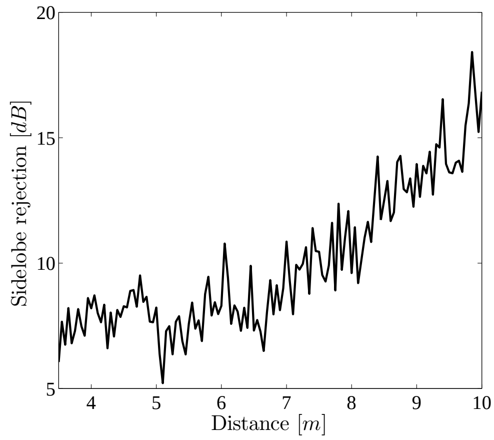

Figure 7: Sidelobe rejection for different distances of the virtual sources w.r.t. the array center. The beam parameters are $\theta = 0°$ and $\phi = 10°$

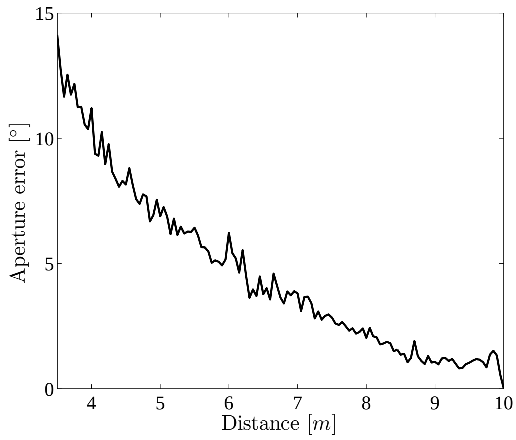

Figure 8: Aperture angle error for different distances of the virtual sources w.r.t. the array center. The beam parameters are $\theta = 0°$ and $\phi = 10°$

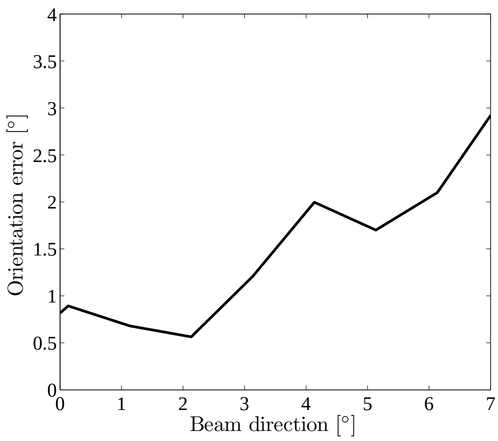

Figure 9: Orientation error for different values of the beam direction θ. The distance of the virtual source from the array center is fixed to $l = 2.7\ m$ and the aperture angle $\phi = 30°$.

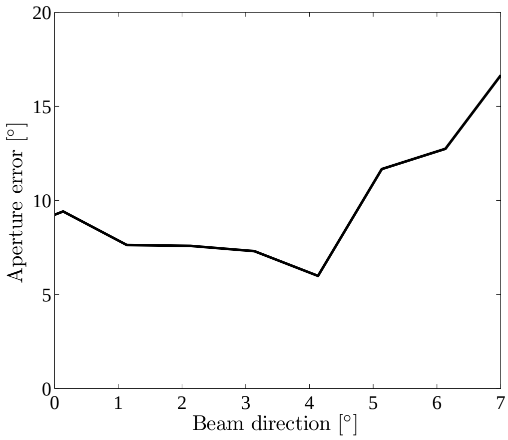

Figure 10: Aperture angle error for different values of the beam direction θ. The distance of the virtual source from the array center is fixed to $l = 2.7\ m$ and the aperture angle $\phi = 30°$.

crease. Figure 11 shows the reason for this behavior: for $\phi = 30°$ and $\theta \geq 5°$, a part of the beam falls outside the listening area and consequently the rendering fails. We have shown this experiment since it represents a critical issue of our system. However we remark that:

- when the maximum of the beam falls outside the rendering area, the rendering of the beam can be advantageously neglected, as it is not relevant in the perception of the acoustics of the environment;
- some preliminary experiments suggest that, when we are working with more than one virtual source, a masking effect appears: the errors introduced by the soundfield of a virtual source are compensated by the other virtual sources active in the area.

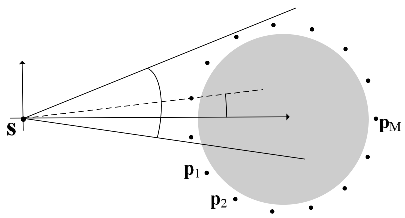

Figure 11: With reference to Figure 10, we observe that when $l = 2.7\ m$, $\phi = 30°$ and $\theta \geq 5°$ part of beam falls outside the listening area and the rendering of the beam fails.

5. CONCLUSIONS AND FUTURE WORKS

In this paper we have shown an efficient technique to perform the rendering of an acoustic beam with an arbitrarily shaped loudspeaker array. The position of the virtual source, the aperture of the beam and its orientation can be arbitrarily controlled. We have shown with some experimental results the feasibility and the critical issues of the approach.

We are currently extending our work into two directions. First we aim to deal with broadband sources (e.g. speech signals): promising results suggest us to investigate on the possibility of computing solutions of eq.(8) for a set of constrained frequencies. Loudspeakers filters are then determined through parabolic interpolation of the amplitudes and cubic interpolation of the phases of the loudspeaker weights at the specified constrained frequencies. We are also investigating a technique for the rendering of a virtual environment based on a geometric decomposition of the wavefield into multiple elementary acoustic beams. The beams are generated from a set of virtual sources whose positions, as well as the beam orientations and apertures, are compliant with the acoustics of the virtual environments. The complex weights vector associated to the emitters are calculated for each beam, and the results are summed up to obtain the global coefficient vector that allows to render the desired soundfield.

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