DIGITAL SIMULATION OF "BRASSINESS" AND AMPLITUDE-DEPENDENT PROPAGATION SPEED IN WIND INSTRUMENTS

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ABSTRACT

The speed of sound in air increases with pressure, causing pressure peaks to travel faster than troughs, and leading to a sharpening of the propagating pressure waveform. Here, this nonlinear effect is explored, and its application to brass instrument synthesis and its use as an audio effect are described. Acoustic measurements on tubes and brass instruments are presented showing significant spectral enrichment, sometimes referred to as "brassiness." The effect may be implemented as an amplitude-dependent delay, distributed across a cascade of incremental delays. A bidirectional waveguide, having a pressure-dependent delay, appropriate for musical instrument synthesis, is presented. A computationally efficient lumped-element processor is also presented. Example brass instrument recordings, originally played softly, are spectrally enriched or "brassified" to simulate a fortissimo playing level.

1. INTRODUCTION

Conventional linear analysis of acoustic wave propagation assumes that the speed of sound is essentially constant in the air medium, and digital simulations of musical wind instruments usually incorporate the same assumption [1, pp. 11-12]. The memory buffer representing a linear propagation medium produces a time delay that is independent of the signal amplitude. Of the various nonlinearities producing amplitude-dependent spectral brightening in a brass instrument, only the pressure-controlled valve in the excitation and feedback path is commonly implemented. One exception is [2], in which an amplitude-dependent delay was used in a waveguide algorithm for synthesizing brass sounds.

While the assumption of constant propagation speed is a valid approximation at moderate sound pressure levels, it becomes unrealistic at the high levels that occur inside musical instruments such as the trombone and trumpet, which can exceed 160 dB [3, 4]. The high-pressure peaks of an acoustic waveform travel faster than the low-pressure troughs [5]. These propagation velocity differences lead to progressive waveform distortion (as illustrated in Figure 1), increasing high-frequency content and— at high pressure levels and long acoustic path lengths— shock waves with impulsive pressure transitions. Musical acousticians have documented the occurrence of both shock waves [4] and sub-shock spectral enrichment [6] in brass instruments, including the trombone.

Amplitude-dependent wave propagation speed can be modeled in terms of the acoustic wave equation, and digitally simulated using finite element methods [7]. As developed below, the pressure-dependent sound speed has the effect of a level-dependent time delay on traveling waves. Tassart, et al. [8] described this phenomenon in the context of acoustic waves and digital waveguide simulations. Valimaki, et al. [9] described the application of signal-dependent nonlinearities to physical models using fractional-delay filters. Tolonen, et al. [10] proposed an amplitude-dependent time delay to model the increase in pitch with waveform amplitude on a vibrating string. In this work, we explore both distributed and lumped implementations of an amplitude-dependent sound speed. The focus here is on "physically informed" sonic modeling [11], suitable for digital audio effects, rather than precise conformity with the acoustical physics of musical instruments. The resulting algorithms are suitable for real-time digital processing with relatively low computational complexity.

Signal delays varied at audio rates are capable of producing spectrally rich sounds. In one example, Stilson [12] modulated the delay in a Karplus-Strong string model with a sinusoid having a frequency near the string fundamental. In another example [13], sounds having both FM and AM characteristics were generated by modulating the coefficients of spectral delay filters at audio rates. Associating the varying time delay with the instantaneous amplitude of an input signal produces a brightening of the spectrum similar in character to that of a brass instrument [3]. Accordingly, the term "brassification" is used here to describe the process of delaying a signal according to its amplitude.

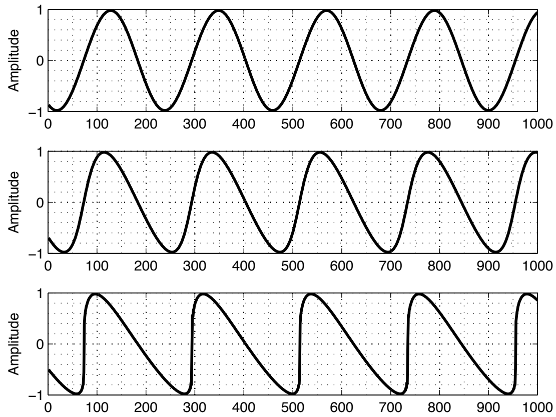

Figure 1: *Waveforms of sinusoidal pressure waves at low, medium and high pressure levels, after propagating the same distance.*

In Section 2 below, the nonlinear wave equation incorporating a pressure-dependent sound speed is explored, and acoustic measurements showing waveform sharpening are presented. Section 3 discusses discretization of the nonlinear wave equation. Implementations of bidirectional waveguide sections producing an amplitude-dependent propagation time and aimed at brass instrument synthesis applications are described in section 4. Audio effects architectures employing an amplitude-dependent delay, and including equalized and side-chain structures, are presented in Section 5. Finally, Section 6 concludes the paper.

2. AMPLITUDE-DEPENDENT WAVE PROPAGATION

2.1. Acoustic Wave Propagation and Waveform Sharpening

The one-dimensional acoustic wave equation describes the behavior of air pressure fluctuations $p(x, t)$ along position x and time t,

$$\frac{\partial^2 p}{\partial t^2} = v^2(x,t)\frac{\partial^2 p}{\partial x^2} \tag{1}$$

where $v(x, t)$ represents the speed of sound [5]. The wave equation (1) propagates disturbances along the x axis with speed $v(x,t)$, which depends weakly on the local, instantaneous air pressure,

$$v(x,t) = c_0 + \beta\frac{p(x,t)}{P_0} \tag{2}$$

where c_0 represents the small-signal speed of sound in air, P_0 the undisturbed air pressure, and β the coefficient of nonlinearity.

The quantity β/P_0 is positive, causing sound wave peaks to travel faster than troughs. This effect progressively distorts the propagating waveform, as illustrated in Figure 1, sharpening the transitions between successive low-pressure and high-pressure portions of the waveform. A sound that starts off as a sinusoid will acquire more of a sawtooth shape, and an increasing amount of high-frequency content. If the product of the sound amplitude and the distance traveled becomes sufficiently large, the slower moving trough would seemingly be overtaken by the preceding peak. What in fact happens, however, is that a shock forms—an abrupt, nearly instantaneous, transition between trough and peak.

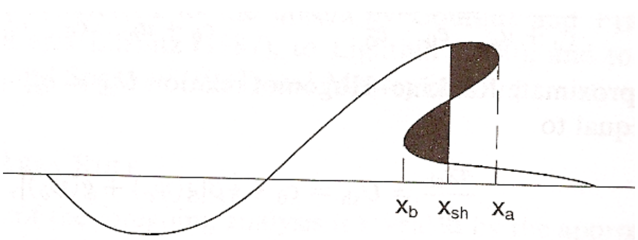

Figure 2: *Multivalued (non-physical) spatial waveform at a fixed time that would seemingly result from a peak overtaking a trough. Instead, a shock transition is formed. From [5], p. 104.*

The shock appears near the location at which the areas of the fast-propagating peak ahead of the shock and the slow-propagating trough behind the shock are balanced; this point is marked by x_{sh} in the example of Figure 2. It turns out that loss mechanisms in air, not included in equation (1), lead to the dissipation of a shock wave after its formation [5].

As an example of the magnitudes involved, consider a sound of 140 dB SPL, which is intense but well below the maximum levels measured in musical instruments such as the trombone [4,6]. This level corresponds to a peak-to-peak pressure fluctuation of 679 Pa, which is nearly 0.7 percent of the static atmospheric pressure of 10^5 Pa. In equation (2), the physical quantity β for air is approximately 1.2 times the nominal speed of sound, so the difference in speed between the peaks and the troughs of the waveform is approximately 0.814 percent of the average or small-signal speed. Over a propagation distance of 2 m (typical for the trombone), this speed difference between peak and trough leads to an arrival-time difference of 48.7 microseconds, which is approximately one-fourth of the period of a 5 kHz signal. This corresponds to a significant steepening of the waveform and brightening of the spectrum.

2.2 Acoustic Measurements

To confirm and quantify the occurrence of an amplitude-dependent propagation velocity at sound levels and path lengths corresponding to those inside a trumpet or trombone, we attached a compression-driver loudspeaker (Atlas PD-30, 30 W) to a PVC plastic tube with inside diameter of 1.27 cm and length 3 m. A microphone was placed inside the tube 2 m from the source, a distance roughly corresponding to the acoustic path length of a trombone. Windowed bursts of several cycles of a 2.205 kHz sine wave were applied to the loudspeaker using a range of amplitudes. The received microphone signals, seen in Figure 3(a), and normalized to have unit amplitude in Figure 3(b), show noticeable waveform sharpening, with their peaks traveling faster than their troughs in a manner well approximated by equation (2). The propagating waveforms experience a high-frequency enrichment, even at signal levels and path distances well below those required for shock formation.

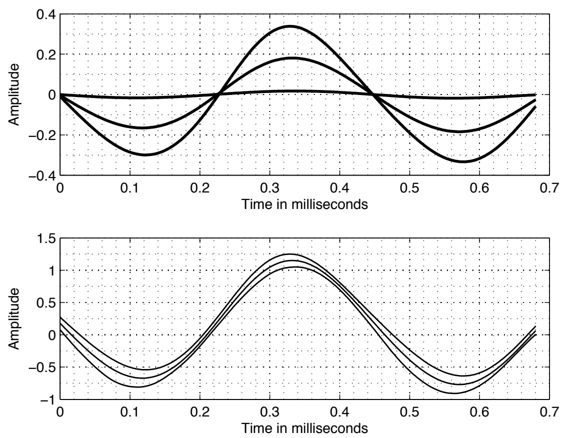

Figure 3: *Acoustic waveforms of windowed sinusoidal bursts at three different pressure levels (top) and the same signals normalized to the same level and slightly offset vertically for clarity (bottom).*

3. WAVE PROPAGATION SIMULATION

3.1 Wave Equation Discretization

While the wave equation (1) provides a general description of the behavior of the disturbances propagating in the medium, we will find it useful to separately consider left- and right-traveling waves, governed by the first-order equation pair

$$\frac{\partial p}{\partial t} \pm (c_0 + \beta \frac{p}{P_0}) \frac{\partial p}{\partial x} = 0 \quad . \tag{3}$$

Given a pressure $g(x)$, defined along the x axis at time t=0, and a wave propagating in the +x direction

$$p(x,t) = g\left(x - (c_0 + \beta \frac{p(x,t)}{P_0})t \right) \quad . \tag{4}$$

Assuming β=0, corresponding to an ideal linear medium, the initial waveform $g(x)$ is seen to travel down the x axis with speed c_0. After a time Δt, the waveform is simply translated , intact, a distance $c_0\ \Delta t$. By contrast, if β is positive, as it is for an air medium, the waveform evolves as it propagates: the peaks travel faster than the troughs.

For a pressure $h(t)$, defined for all time at the position x=0, the pressure propagating along the $+x$ axis satisfies

$$p(x,t) = h\left(t - \frac{x}{c_0 + \beta \frac{p(x,t)}{P_0}} \right) \quad . \tag{5}$$

For β=0, the pressure noted at x_0>0 is simply the pressure at x=0, delayed by x_0/c_0. In the presence of β>0, the pressure at x_0 is approximately the pressure amplitude at x=0, delayed according to its value, with peaks arriving relatively sooner than troughs.

The fact that the nonlinear wave equation implies a pressure-dependent time delay can be seen via a simple discretization of (4). Consider a delay line having sample locations labeled by n, and containing a pressure waveform $p(n,t)$ at time t. Assume that the small-signal sound speed c_0 is one sample location per sample interval, and approximate the time and position derivatives by first-order differences,

$$\frac{\partial p}{\partial t} = p(n,t) - p(n,t-1), \tag{6}$$

$$\frac{\partial p}{\partial x} = p(n,t-1) - p(n-1,t-1) \quad . \tag{7}$$

A little algebra gives the delay line waveform at time step t in terms of its contents at time step t-1,

$$p(n,t) = \alpha\, p(n,t-1) + (1-\alpha)\, p(n-1,t-1) \tag{8}$$

where

$$\alpha = -\beta \frac{p}{P_0} \quad . \tag{9}$$

Note that when β=0, corresponding to propagation in a linear medium, the waveform is simply shifted intact, one sample position for each new time step. In general, when β is not equal to zero, the waveform at time t and position n is a linear interpolation of the waveform at positions n and n-1 at time t-1. The waveform at time t and position n, therefore, approximates the waveform at time t-1 in the neighborhood of location n-1, just before n-1 for positive pressures and just after n-1 for negative pressures.

A similar discretization gives the pressure at position n as a linear interpolation of its value at position n-1 between times t and t-1,

$$p(n,t) = \eta\, p(n-1,t) + (1-\eta) p(n-1,t-1) \tag{10}$$

where the interpolation coefficient η is

$$\eta = -\frac{\beta \frac{p}{P_0}}{1 + \beta \frac{p}{P_0}} \quad . \tag{11}$$

We again have the interpretation that the pressure at position n is the pressure at position n-1 delayed according to its value.

3.2 Amplitude-Dependent Delay

In view of the interpretation above, the amplitude-dependent propagation may be implemented in discrete time as a cascade of amplitude-dependent elements as shown in Figure 4. A buffer indexed by n contains the propagating waveform. At every time step t, the waveform at position n is replaced by its value at position

$$n - 1 - \beta \frac{p}{P_0}, \tag{12}$$

where a sound speed of c_0 = 1 sample position per sample interval is assumed.

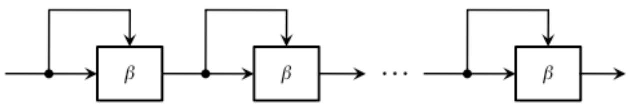

Figure 4: *Discretized unidirectional amplitude-dependent time delay using cascaded elements. The arrows entering the top of each element represent the modulation of the element's delay by the pressure at its input.*

The amplitude-dependent delay elements comprising the cascade in Figure 4 may be implemented in a number of ways [14, 15]. FIR approaches are particularly simple: The input is upsampled, and a low-order interpolation applied. Linear interpolation according to (8) or (10), or fourth-order Lagrange interpolation works well with modest upsampling factors such as two or four. The high-frequency droop present in the FIR interpolation is not unwelcome, as it is in some sense similar to damping mechanisms present in air, not included in (3), which affect the formation and evolution of shock.

First-order allpass filtering may also be used to implement the needed variable delay. There are, however, some drawbacks to this approach. First, these filters are dispersive, such that when the low frequencies are delayed a little more than one sample, the high frequencies will be delayed a little less than one sample. Second, distortion artifacts may be introduced by audio rate modulation of allpass coefficients, although this too can be controlled to some extent [13, 16]. Both the underlying mathematics and the available implementation options are generally analogous to those described in [10] for modeling tension-dependent nonlinearity in plucked strings. Overall, we suggest using upsampling with a low-order FIR interpolator to implement the desired amplitude-dependent delay element.

4. MUSICAL INSTRUMENT SOUND SYNTHESIS

4.1 Bidirectional waveguide

The discretization above may be configured to implement a bidirectional waveguide and used to simulate the bore of a wind instrument. To do so, two variable delay lines propagating signals in opposite directions are used. In an instrument bore, however, the sound speed depends on the total bore pressure, rather than the individual left- and right-traveling pressures. The suggested waveguide implementation is shown in Figure 5. At every position along the bidirectional waveguide, the sum of the left- and right-going pressures is used to modulate the respective variable delays.

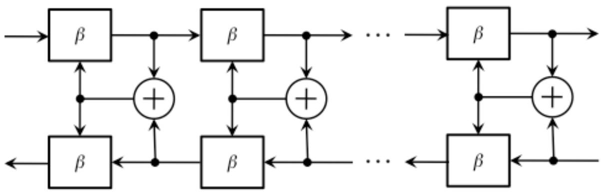

Figure 5: *Modeling bidirectional wave propagation with amplitude-dependent delay elements*

4.2 Lumped-element Simplification

The complexity of the waveguide shown in Figure 5 may be reduced by approximating the effect of a number n of cascaded variable delays into a single combined variable delay of value n times $(1+\beta p/P_0)$. Using lumped delay elements, the left- and right-going waves can be summed at a sparse set of locations along the acoustic tube to provide a delay control signal that includes the effect of the interaction between the outgoing and reflected waves. Only a limited number of delay locations are required, because the pressure wave inside the bore of a brass instrument is dominated by low frequencies [6], corresponding to low spatial frequencies. Alternatively, a unidirectional implementation of the variable delays may be considered sufficient, in view of the fact that the high-frequency signal components, which are the ones primarily affected by delay-time modulation, are largely transmitted through the instrument bell rather than being reflected back into the bore. On the other hand, in detailed digital simulation of the trombone for synthesis appplications [17], it was shown that including the backward wave within the oscillatory feedback loop does affect the fundamental frequency and increase the brassiness of the synthesized output signal.

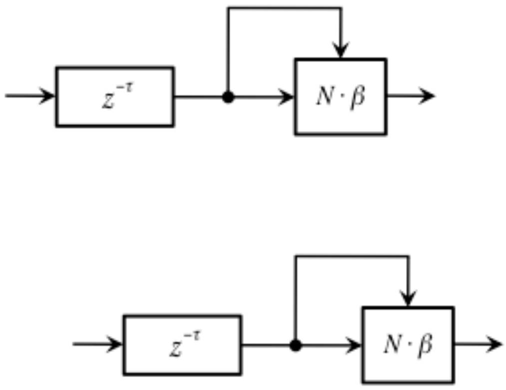

Figure 6: *Commuting a single large amplitude-dependent delay to the end of a fixed delay.*

5. AUDIO EFFECTS

5.1 Lumped "Brassification"

In the context of applying amplitude-dependent delay to existing audio signals (rather than de novo synthesis of musical sounds), there is little reason to model the detailed physics by implementing distributed variable delays. A natural approach is to lump the delay processing into a single amplitude-dependent delay line, as shown in Figure 6. Each new arriving signal sample, after upsampling to a higher rate, can be written (added) into the appropriate positions in the delay line using linear (or Lagrange) interpolation.

To scrupulously model the physics of acoustic wave propagation, including shock fronts at high amplitudes, any samples that are computed to be overtaken by earlier higher-amplitude samples should be discarded, as described in Section 2.1. This logic can be implemented in the interpolated delay line. We have found experimentally, however, that omitting this feature (and instead interpolating and adding every input sample into the delay line even if it arrives "late") results in processed sounds with a brighter spectrum that, in the opinion of the authors, sounds more musically appealing. This may be attributable to the fact that acoustic shock waves correspond to energy loss, and that a larger amount of time-delay modulation can be applied to the signal when the shock-wave feature is omitted.

When the shock-wave feature is omitted from the delay-line implementation, the amplitude-dependent time-delay becomes equivalent to phase or frequency modulation (PM or FM). The high-frequency harmonics of a wind instrument sound may be regarded as "carrier" signals that are phase-modulated by the dominant low-frequency pressure wave inside the bore of the instrument. Phase modulation of a single carrier frequency produces sidebands that generally increase in prominence as the phase excursion increases, but the relationship is not monotonic. Instead, according to standard PM and FM theory [18], the amplitude of each generated sideband is proportional to an oscillatory Bessel function, resulting in a complex, smooth and musically appealing variation in the spectrum. It is perhaps significant that FM synthesizers are considered particularly successful when emulating brass instruments.

5.2 Audio Effects Architectures

Since the acoustic signal inside the bore of a musical instrument consists primarily of low frequencies [6], especially the fundamental, it is appropriate to filter the signal so as to emphasize its low-frequency content. In this way, the digitally simulated pressure waveform for modulating the time delay will correspond more closely to that inside the instrument. After the amplitude-dependent propagation delay is applied, this filtering should be compensated by a complementary filter emphasizing high frequencies and corresponding, for instance, to the frequency-selective transmission through the flared bell of the horn.

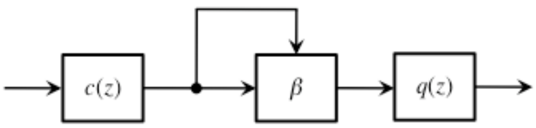

Figure 7: *Equalized "brassifier" with conditioning filter before the amplitude-dependent delay and equalization filter after it.*

This architecture—an equalized "brassifier"—is shown in Figure 7. The filter $c(z)$ conditions the signal to emphasize those features of the signal important for controlling the brassification. The filter $q(z)$ provides complementary equalization chosen to be the inverse of $c(z)$. With conditioning and equalization filters arranged in this manner, low-amplitude signals will pass through the process unchanged, while high-amplitude signals will be brassified (spectrally brightened).

Another brassifier architecture provides a side-chain signal for controlling the delay modification, analogous to the generalized time-varying fractional delay used in [10] to model string tension modulation. As shown in Figure 8, a filtered version of an input signal is used to control the amplitude-dependent delay experienced by the input. This structure, when used with a low-pass side-chain filter, improves the output signal quality of the brassifier while retaining the desired spectral enrichment. Unlike the signal-dependent allpass technique of Kleimola, et al. [13], the use here of upsampling and linear or Lagrange interpolation permits very large amounts of side-chain modulation to be applied without causing excessive aliasing or other undesired distortion.

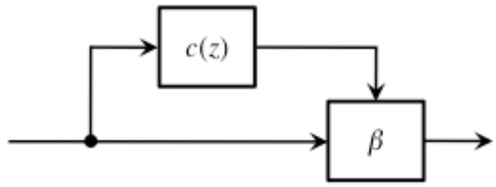

Figure 8: *"Brassifier" with side chain for modulation signal. The filter c(z) conditions the input signal, typically by emphasizing low frequencies, to produce a signal suitable for modulating β, the amplitude-dependent time delay.*

An example of the spectra resulting from using the side-chain architecture to process a recorded trumpet signal is shown in Figure 9. The sound files corresponding to these spectra and related examples are available on the World Wide Web [19].

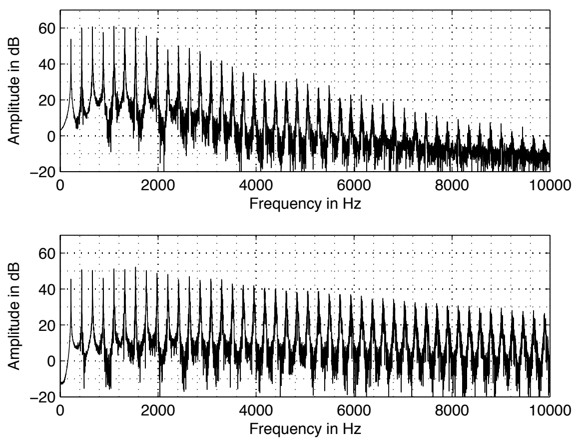

Figure 9: *Spectrum of unmodified trumpet signal (top) and the processed or "brassified" spectrum (bottom).*

6. CONCLUSIONS AND FUTURE WORK

Amplitude-dependent digital signal delay, derived from concepts of nonlinear acoustics, has been shown to produce spectra and sounds that are brass-like in character. The technique can be used for synthesis of musical instrument sounds in physical models, but as implemented here it is especially suitable for modification of pre-recorded or live instrument sounds as a digital audio effect. The digital implementation is capable of modeling the production of acoustic shock waves at high signal amplitudes, but substantial or even increased "brassiness" can be achieved by eliminating shock-wave emulation (i.e., the discarding of late-arriving samples) and using a larger amplitude-dependent delay and modulation index. When shock production is eliminated, the effect can best be interpreted and analyzed in terms of phase or frequency modulation.

The suggested implementation—as a single lumped delay line with FIR interpolation—is free of unwanted artifacts such as spectral dispersion, and its low computational complexity is suitable for real-time applications. As a digital audio effect or for sample-based synthesis, the technique can be used to enrich the spectrum and modify the apparent dynamic level of a musical instrument sound. The effectiveness of amplitude-dependent delay in producing or enhancing brassiness suggests that this nonlinear effect is desirable as a fundamental component of physical models for synthesizing brass instrument sounds.

Future work will include the application of this technique to non-brass instrument sounds. Another anticipated extension is the use of multi-band processing so that a modulation index large enough to produce audibly significant PM sidebands can be applied even to low-frequency signal components.

7. ACKNOWLEDGMENTS

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