A PARAMETRIC MODEL OF PIANO TUNING

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ABSTRACT

A parametric model of aural tuning of acoustic pianos is presented in this paper. From a few parameters, a whole tessitura model is obtained, that can be applied to any kind of pianos. Because the tuning of piano is strongly linked to the inharmonicity of its strings, a 2-parameter model for the inharmonicity coefficient along the keyboard is introduced. Constrained by piano string design considerations, its estimation requires only a few notes in the bass range. Then, from tuning rules, we propose a 4-parameter model for the fundamental frequency evolution on the whole tessitura, taking into account the model of the inhamonicity coefficient. The global model is applied to 5 different pianos (4 grand pianos and 1 upright piano) to control the quality of the tuning. Besides the generation of tuning reference curves for non-professional tuners, potential applications could include the parametrization of synthesizers, or its use in transcription / source separation algorithm as a physical constraint to increase robustness.

1. INTRODUCTION

One of the main factor that makes piano tuning so distinctive is the inharmonic nature of piano tones [1]. For a perfectly soft string, the spectrum of a note sound should be composed of purely harmonic partials. In practice, because of the stiffness of the piano wire, each partial is slightly sharper, and the higher the rank of the partial, the sharper the partial. This phenomenon directly affects the tuning because it constraints the tuner to stretch the intervals in order to cancel or control beats. Moreover, psycho-acoustical effects seem to be involved in the choice of the amount of stretching that is optimal according to the position in the tessitura [1] [2]. Due to the variations in piano scale designs and tuners' specific techniques, no single standard tuning rule can be established. However, some studies (see [3], [4]) have tried to formalize these rules used by tuners, to approximate aural tuning in a given range of the piano, taking into account inharmonicity measurements.

The purpose of this paper is to simulate aural tuning on the whole tessitura of a particular piano, based on the recordings of only a few isolated notes. This problem can be seen as an interpolation of inharmonicity and fundamental frequency across the whole tessitura, based on a limited set of initial data. In order to get a robust method, we constrain the interpolation with prior information on piano string design and tuning rules. This model can be used to generate tuning reference curves for non-professional tuners, to parametrize piano synthesizers, or be included as a constraint in transcription / source separation algorithms.

In Section 2, physical assumptions used to model the piano string vibration are given, and the relations between piano string design and tuning are discussed. From this considerations, we propose in Section 3 a simple model with 2 parameters to represent the evolution of the inharmonicity coefficient on the whole tessitura. Then, in Section 4, we introduce a 4-parameter model based on tuning rules to generate reference tuning curves, by taking into account the inharmonicity model. We conclude (Section 5) with a discussion on potential applications of such model. For the sake of completeness, we describe in Appendix A the inharmonicity / fundamental frequency estimation algorithm that we used on single note recordings to obtain the reference values.

2. PHYSICAL CONSIDERATIONS IN PIANO TUNING

2.1. Physical modelling of piano string

Solving the transverse wave equation for a plain stiff string with fixed endpoints yields the following modal frequencies [1]:

$$f_n = nF_0\sqrt{1+Bn^2}, \quad n \in \mathbb{N}^+ \tag{1}$$

where n is the mode index or partial rank, B the inharmonicity coefficient, and F_0 the fundamental frequency of a flexible string (with no stiffness). F_0 is related to the speaking length of the string L, the tension T and the linear mass μ according to:

$$F_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{2}$$

Note that $F_0 \approx f_1$, but that strictly speaking this fundamental frequency is not directly measured as one peak in the spectrum: it is a global value of the tone that must theoretically be obtained from the whole set of partials. The stiffness is taken into account in B, with:

$$B = \frac{\pi^3 E d^4}{64\, T L^2} \tag{3}$$

where E is the Young's modulus and d the diameter of the plain string. Note that this model is given for a string with fixed endpoints. It does not take into account the bridge coupling (with finite admittance), which modifies the partial frequencies, mainly in the low frequency domain [5], [6], [1], [7].

2.2. String set design influence on B

Piano strings are designed with the constraint to minimize the discontinuities in physical parameters variations [8], [9]. Three main discontinuities appear along the keyboard: the bass break between the bass and treble bridges, the transition between plain and wrapped strings and the transitions between adjacent keys having different number of strings. The variations of B along the keyboard are mainly affected by the bass break which results in two main trends:

On the treble bridge, from C8 note downwards, B is decreasing because of the increase of L. Down to middle C (C4 note), the values of B are roughly the same for all the pianos and B follows a straight line in logarithmic scale [10]. This result is mainly due to the fact that string design in this range is standardized since it is not constrained by the limitation of the piano size.

To keep a reasonable size of the instrument, the bass bridge design reduces the growth of L. Then the linear mass of the string is increased in order to adjust the value of F_0 according to equation (2). Instead of increasing only the diameter d, which increases B and decreases the breaking strength, the strings are wrapped. Thus, on bass bridge, B is increasing from sharpest notes downwards. Note that the number of keys associated to the bass bridge and the design of their strings are specific for each piano.

2.3. Tuning influence on (F_0,B)

Most of the parameters in equations (2) and (3) are fixed at the string design. The only parameter the tuner can vary in order to adjust F_0 is the tension of the string T. In the same time, T affects the value of the inharmonicity constant B. Consequently, F_0 and B are dependent on each other because of physical relations and tuning considerations. In this paper, we assume that the relative variation of T during the tuning (of an initially slightly detuned piano) is small enough to consider that B remains constant.[1] It allows us to first extract a parametric model for B along the keyboard, and then to deduce tuning reference curves.

3. WHOLE TESSITURA MODEL FOR B

3.1. Parametric model

According to subsection 2.2, B should be modelled by two distinct functions corresponding to the two bridges, and could present a discontinuity at the bass break. In this paper we propose a "continuous" additive model on the whole tessitura, discretized for $m \in [21, 108]$, the midi note index from A0 to C8. We denote it by $B_\theta(m)$, with θ the set of parameters.

Usually, the evolution of B along the keyboard is depicted in logarithmic scale and presents two linear asymptotes. We denote by $b_T(m)$ (resp. $b_B(m)$) the Treble bridge (resp. the Bass bridge) asymptote of $\log B_\theta(m)$. Each asymptote is parametrized by its slope and its Y-intercept.

$$\begin{cases} b_T(m) = s_T \cdot m + y_T \\ b_B(m) = s_B \cdot m + y_B \end{cases} \tag{4}$$

According to [10], $b_T(m)$ is similar for all the pianos so s_T and y_T are fixed parameters. Then, the set of free (piano dependent) parameters reduces to $\theta = \{s_B, y_B\}$. $B_\theta(m)$ is set as the sum of the contributions of these two curves (4) in the linear scale:

$$B_\theta(m) = e^{b_B(m)} + e^{b_T(m)} \tag{5}$$

It should be emphasized that this additivity does not arise from physical considerations, but it is the simplest model that smoothes discontinuities between the bridges. Experimental data will show that it actually describes well the variations of B in the transition region around the two bridges. An example of this model for $B_\theta(m)$ is shown on Figure 1.

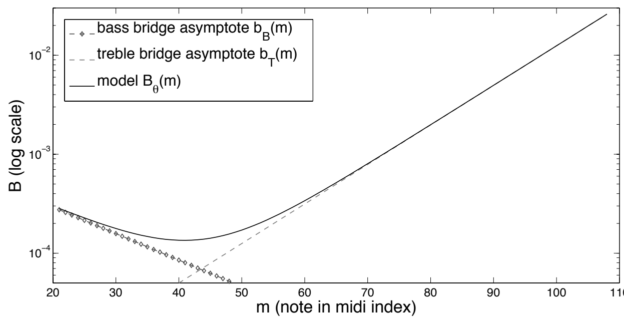

Figure 1: *Model of B along the keyboard.*

3.2. Parameter estimation

The results in this paper are obtained from single note recordings of 3 separate databases: Iowa[2] (1 grand piano), RWC [11] (3 grand pianos) and MAPS[3] (1 upright piano). For a given note index $m \in [21, 108]$, $F_0^*(m)$ and $B^*(m)$ are the estimated values of $F_0(m)$ and $B(m)$ using the algorithm described in Appendix A.

We first estimate the fixed parameters $\{s_T, y_T\}$ using the data of all the pianos in the range C4-C8 ($m \in [60, 108]$, the standardized design range). These are obtained by a L1 linear regression (to reduce the influence of potential outliers) on the average of the estimated inharmonicity curves in logarithmic scale over the different pianos. We find $s_T \simeq 9.26 \cdot 10^{-2}$, $y_T \simeq -13.64$. These results are in accordance with estimates based on physical considerations [10]: $s_{T[10]} \simeq 9.44 \cdot 10^{-2}$, $y_{T[10]} \simeq -13.68$.

Finally, each piano is studied independently to estimate the particular parameters $\theta = \{s_B, y_B\}$ on a set of few notes M. θ is estimated minimizing the L1 distance between $B^*(m)$ and $B_\theta(m)$:

$$\theta^* = \arg\min_\theta \sum_{m \in M} |B^*(m) - B_\theta(m)| \tag{6}$$

We present on Figure 2 the curves of $B_\theta(m)$ obtained for every piano from a set of 3 quasi-equally spaced notes taken in the bass range A0-D3 ($m \in [21, 50]$). The discontinuity of the bass break is clearly observable for some pianos on the reference data curves (for instance between C#2 ($m = 37$) and D2 ($m = 38$) notes for the 2nd grand piano of the RWC database) and does not always occur at the same keys. The global variations are well respected. Note that some outliers are present (in the high treble range) in the whole tessitura data curves. This problem is discussed in the appendix and is due to the fact that for sharpest notes the partials are not in sufficient number to have a robust estimation of B. To evaluate the distance between the model and the whole tessitura data those outliers have been manually removed before the computation of the relative deviation between $B^*(m)$ and $B_\theta(m)$. We present on Figure 3 the histograms of the relative deviation computed in

[1] For instance, if a note is increased by a quarter tone (50 cents) during the tuning, $\frac{\Delta F_0}{F_0} \simeq 2.9\%$. According to equations (2) and (3), $\frac{\Delta B}{B} = -2 \cdot \frac{\Delta F_0}{F_0} \simeq 5.9\%$.

[2] http://theremin.music.uiowa.edu

[3] http://www.tsi.telecom-paristech.fr/aao/en/category/database/

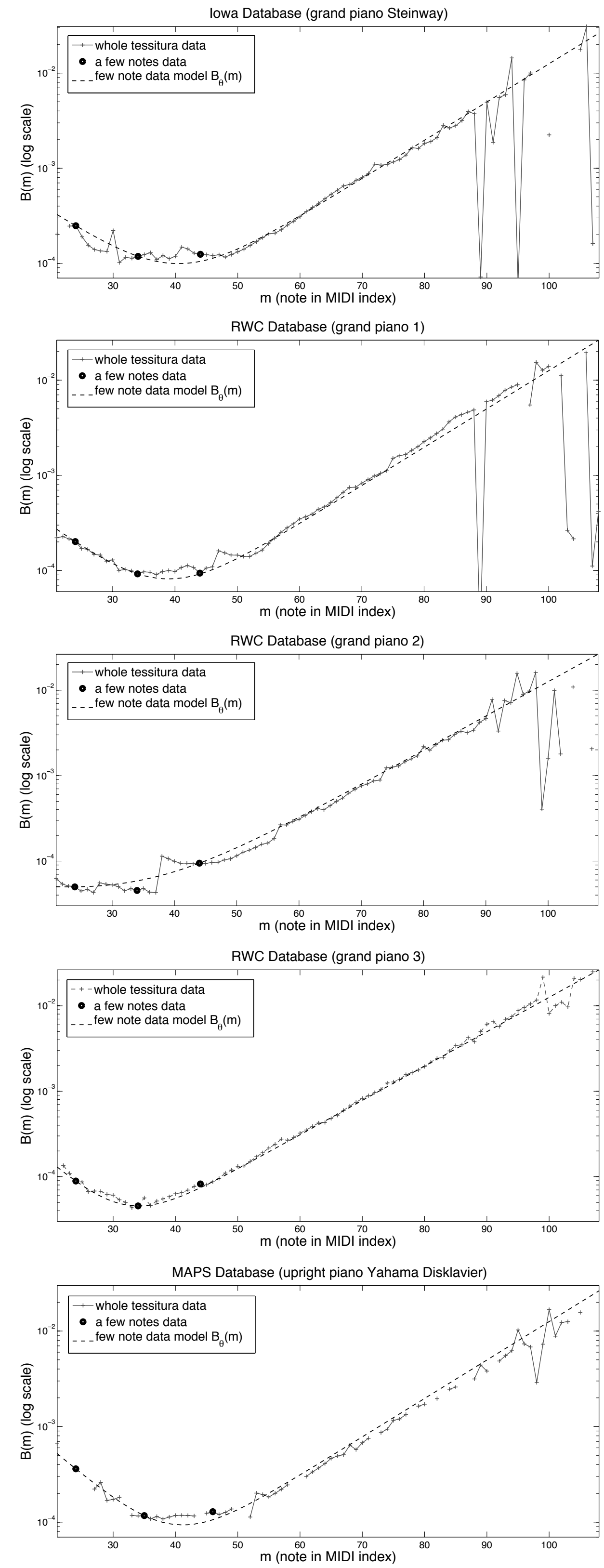

Figure 2: *Superposition of the model $B_\theta(m)$ (estimated from 3 notes in the bass range) with the whole tessitura reference values for 5 different pianos.*

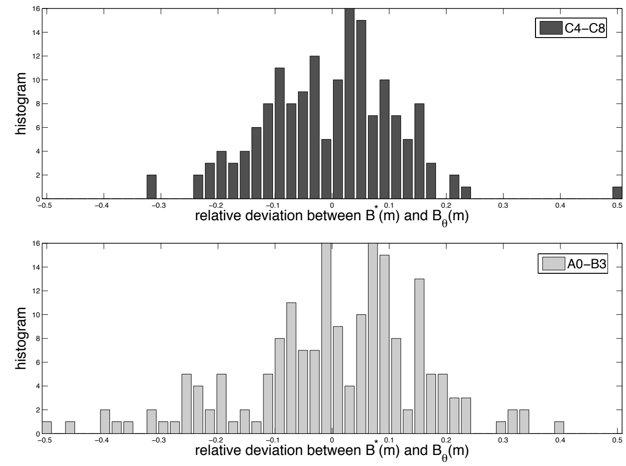

Figure 3: *Histogram of the relative deviation between $B^*(m)$ and $B_\theta(m)$ computed for all the pianos in the range C4-C8 (at the top) and A0-B3 (at the bottom).*

A0-B4 ($m \in [21, 71]$) and C4-C8 ($m \in [72, 108]$) ranges. In C4-C8 range, the mean and the standard deviation are respectively equal to $-4.2 \cdot 10^{-3}$ and $1.16 \cdot 10^{-1}$. In A0-B4 range we have respectively $4.6 \cdot 10^{-3}$ and $1.57 \cdot 10^{-1}$.

4. WHOLE TESSITURA MODEL FOR F_0

4.1. Aural piano tuning principles

Every tuning begins by the tuning of the reference note, in most cases the A4 at 440Hz. To do so, the tuner adjusts the tension of the strings to cancel the beats produced by the difference of frequency of the tuning fork (a quasi perfect sinusoid) and the first partial of the note. Thus, $f_1(m = 69) = 440$Hz.

From A4, the tuner builds the reference octave F3-F4 according to the equal temperament in controlling (or counting) the beats of different intervals (for instance between the 3rd partial of a reference note and the 2nd of its fifth) [12]. For high inharmonicity pianos the frequency deviation between the first partial of the note and the theoretical fundamental frequency given by the equal temperament in this octave can be about ± 8.6 cents ($\pm 0.5\%$) [3].

Finally, from this reference octave in the middle of the tessitura, each note is tuned step by step with the same procedure. Because of the partial deviation due to the inharmonicity, the octaves are stretched to more than a 2:1 frequency ratio. For a reference note of midi index m, $f_1(m + 12) > 2f_1(m)$ because $f_2(m) > 2f_1(m)$. Moreover, the amount of stretching of the octaves in the different parts of the keyboard is linked to psychoacoustic effects and tuner's personal tastes. It is well known that the piano sounds better in the bass range if the amount of stretching is more important than in the treble range (even if the inharmonicity effect is less important). This fact is linked to the underlying choice of the type octave during the tuning [2]. For instance, in a 4:2 type octave, the 4th partial of the reference note is matched to the 2nd partial of its octave. Depending on the position in the tessitura, the piano can be tuned according to different type octaves: 2:1, 4:2, 6:3, 8:4, ... or a compromise of two. This means that the tuner may not be focused only on cancelling beats between a pair of partials, but that he controls an average beat generated by a few partials of the two notes.

Here, we propose a 4-parameter model for synthesizing aural tuning on a given piano. The steps may be a simplified version of those done by a tuner but the global considerations (stretching inherent in the inharmonicity and the type octave choice) are taken into account. We begin by the A4 reference note, setting $f_1(m = 69) = 440$Hz. Then, we introduce (Subsection 4.2) a 3-parameter model to estimate the tuning of all the A keys from the A4. In Subsection 4.3, we propose a model to tune all the notes inside of a fixed octave interval (for instance A4-A5 previously determined). Finally, in Section 4.4, we introduce 1 extra parameter to take into account a global detuning and we present the results for the 5 pianos. Note that the following expressions are established for upper interval construction, but the same reasoning can be applied for lower intervals.

4.2. Octave interval tuning

4.2.1. Model

During the tuning of an upper octave interval, the cancellation of the beats produced by the u-th partial of a reference note indexed $m - 12$ and the v-th partial of its octave indexed m ($u = 2v$) can be done by tuning $F_0(m)$ such as[4]:

$$F_0(m) = F_0(m-12) \cdot \frac{u\sqrt{1 + B(m-12)u^2}}{v\sqrt{1 + B(m)v^2}} \tag{7}$$

The choice of the type octave is parametrized by introducing the variable $\rho \in \mathbb{N}^+$, such as $u = 2\rho$ and $v = \rho$. Usually the maximal value for ρ is 6 (it corresponds to a 12:6 type octave which can sometimes occur in the low bass range of grand pianos). We denote by $\rho_\varphi(m)$ the model of ρ on the whole tessitura given for a set of parameter φ. Then,

$$F_0(m) = 2\, F_0(m-12)\sqrt{\frac{1 + B(m-12) \cdot 4\rho_\varphi(m)^2}{1 + B(m) \cdot \rho_\varphi(m)^2}} \tag{8}$$

This model takes into account the cancellation of the beats produced by a single pair of partials. In practice, the deviation $\frac{F_0(m)}{2F_0(m-12)}$ should be a weighted sum of the contribution of two pairs of partials, because the amount of stretching may result from a compromise between two type octaves. An alternative model to take into account this weighting is to allow non-integer values for $\rho_\varphi(m) \in [1, +\infty[$. For example, if the octave tuning of a note indexed m is a compromise between a 2:1 and 4:2 type octaves, $\rho_\varphi(m)$ will be in the interval $[1, 2]$. This model looses the physical meaning ($u = 2\rho$ and $v = \rho$ are not anymore related to partial ranks), but presents the advantage to be easily inverted to estimate $\rho_\varphi(m)$. Note that this model for octave interval tuning could be generalized to other intervals tuning by considering the beats inherent in the equal temperament. Indeed, in equal temperament only octave intervals can have consonant partials.

[4]Note that F_0 is defined as being the fundamental frequency for a perfectly soft string. In practice it is not present in the piano tone so the tuner adjusts f_1, the frequency of the first partial . F_0 is used in the equations of this section because it is more practical to manipulate. In the end, equation (1) is applied to obtain $f_1(m) = F_0(m)\sqrt{1 + B(m)}$.

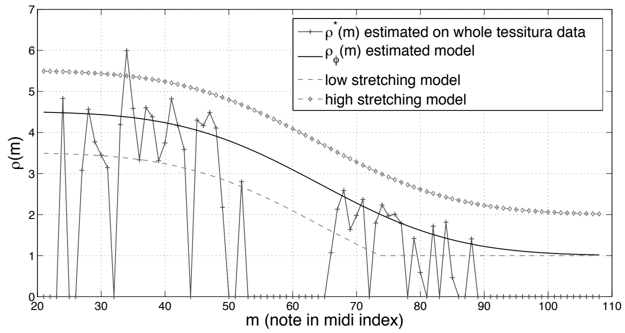

Figure 4: *Superposition of $\rho^*(m)$ averaged from 3 pianos, and the model $\rho_\varphi(m)$. High and low stretching curves are respectively, arbitrarily defined by $\rho_\varphi(m) + 1$ and max$(\rho_\varphi(m) - 1, 1)$.*

4.2.2. Estimation of $\rho_\varphi(m)$

We choose to model $\rho_\varphi(m)$ as follows:

$$\rho_\varphi(m) = \frac{K}{2} \cdot \left(1 - \text{erf}\left(\frac{m - m_0}{\alpha}\right)\right) + 1 \tag{9}$$

with erf the error function. It expresses the fact that the amount of stretching inherent in the type octave choice is decreasing from the low bass range to the high treble range and that it is limited by horizontal asymptotes at each extremity. The set of parameters is then $\varphi = \{m_0, \alpha, K\}$. m_0 is a parameter of translation along m. α rules the slope of the decrease. K settles the value of the low bass asymptote. Note that in (9) the high treble asymptote is set to 1 because it corresponds to the minimal type octave (2:1).

$\rho^*(m)$ is estimated on the data $F_0^*(m)$ and $B^*(m)$ by inverting equation (8):

$$\rho^*(m) = \sqrt{\frac{4F_0^*(m-12)^2 - F_0^*(m)^2}{F_0^*(m)^2 B^*(m) - 16F_0^*(m-12)^2 B^*(m-12)}} \tag{10}$$

Then, the set of parameters is estimated minimizing the L1 distance between $\rho_\varphi(m)$ and $\rho^*(m)$ on a set M of notes.

$$\varphi = \arg\min_{\varphi} \sum_{m \in M} |\rho^*(m) - \rho_\varphi(m)| \tag{11}$$

Finally, $\rho^*(m)$ has been estimated for the 3 best tuned pianos of the database (the selection criterion was that their tuning deviation from equal temperament is following the global variations of the Raylsback theoretical curve [1]), and averaged to obtain a mean curve from different tuners.[5] The parameter estimation gives $m_0 \simeq 64$ (the curve is centred on the middle octave), $\alpha \simeq 24$, and $K \simeq 4.51$ (in the low bass range, the tuning is a compromise between 8:4 and 10:5 type octaves). The results are depicted on Figure 4. Some values of $\rho^*(m)$ are missing in the treble and the bass range because we removed the outliers from the estimation of $B^*(m)$ and $F_0^*(m)$. Because F3-F4 ($m \in [53, 65]$) is the reference octave of the tuning, $\rho^*(m)$ is not estimated on it. From

[5]In practice the estimation of $\rho^*(m)$ could be done for each piano to model their actual tuning. We choose here to obtain a reference stretching curve from well-tuned pianos in order to control the tuning of the 5 pianos in Subsection 4.4. In this case the application is not anymore the interpolation of the tuning on the whole tessitura of each piano.

$\rho_\varphi(m)$ defined as a mean stretching model, we define arbitrarily a high stretching model by $\rho_\varphi(m)+1$ and a low stretching model by $\max(\rho_\varphi(m)-1, 1)$. The low stretching model is saturated to 1 in the treble range because $\rho \in [1, +\infty[$.

4.3. Model for semitone tuning in a given octave interval

Once the octave intervals are built according to equation (8), the whole tessitura is interpolated semitone by semitone. If there was no stretching, the semitones would be equally spaced by the ratio of $\sqrt[12]{2}$ given by the equal temperament. In practice, the frequency ratio between 2 adjacent notes is a little higher than $\sqrt[12]{2}$. We model this deviation as follows:

$$f_1(m+1) = f_1(m)\ \sqrt[12]{2+\varepsilon(m+1)} \tag{12}$$

with $\varepsilon \ll 1$. As a first order model, we assume that ε varies linearly with B. This dependence underlines the fact that the higher B, the higher the deviation should be. Thus,

$$\varepsilon(m+1) = \lambda \cdot B(m+1) \tag{13}$$

λ is estimated in the given octave interval and takes into account the stretching related to the type octave through the previous estimation of $f_1(m+12)$. Recursively we have:

$$f_1(m+12) = f_1(m) \prod_{p=1}^{12} \sqrt[12]{2+\lambda \cdot B(m+p)}$$

By taking the logarithm, and developing at the first order, λ can be estimated by:

$$\lambda = \frac{24 \log\big(f_1(m+12)/2f_1(m)\big)}{\sum\limits_{p=1}^{12} B(m+p)} \tag{14}$$

4.4. Global detuning and results

Once the tuning has been estimated on the whole tessitura, the real piano tuning can present a slight global detuning compared to the model $f_1(m)$. The detuning or deviation of each note from the equal temperament (ET) is given in cents by:

$$d_1(m) = 1200 \log_2 \frac{f_1(m)}{F_{0\mathrm{ET}}(m)} \tag{15}$$

with

$$F_{0\mathrm{ET}}(m) = 440 \cdot 2^{(m-69)/12} \tag{16}$$

We introduce the global detuning through the 4th parameter d_g, which is estimated by minimizing the L1 distance, on the reference octave F3-F4 ($m \in [53, 65]$) between $d_1^*(m)$, the detuning estimated on data, and $d_1(m)+d_g$ the detuning of the model:

$$d_g = \arg\min_{d_g} \sum_{m=53}^{65} \left| d_1^*(m) - (d_1(m)+d_g) \right| \tag{17}$$

Finally, Figure 5 shows the deviation from ET of the estimated models for the 3 amounts of stretching (mean, low and high) applied to the 5 pianos. Comparing the curves of the model and the data, we can see that RWC2 and RWC3 piano seem well-tuned. On the contrary, the tuning of RWC1 piano is not stretched in the bass range and the tuning of Iowa and MAPS pianos should be a little more streched in the treble range, according to our model. Further research and discussions with piano tuners will investigate whether this discrepancy is indeed due to an inappropriate tuning, or a limitation of our model.

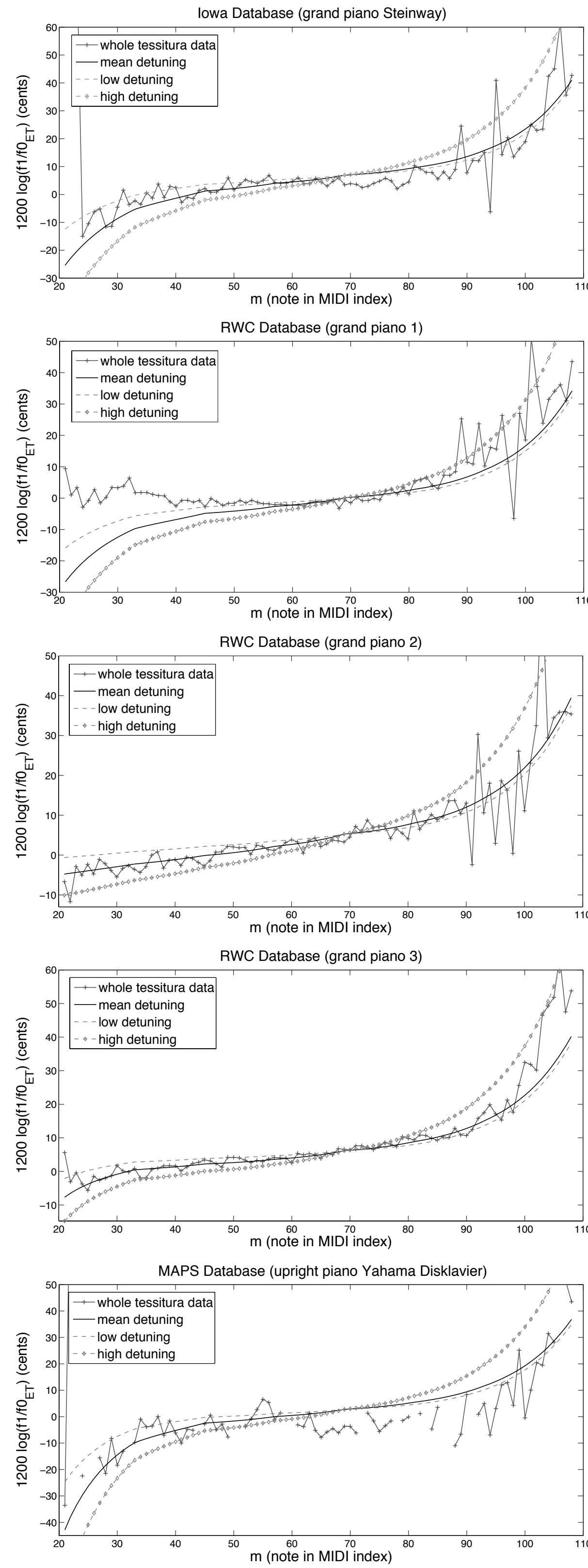

Figure 5: *Superposition of the deviation from ET of the model $f_1(m)$, with the whole tessitura data for 5 pianos. $f_1(m)$ is computed for the 3 models of $\rho_\varphi(m)$ depicted on the figure 4 and corresponding to a mean, low and high octave stretching.*

5. CONCLUSIONS

We presented in this paper a model composed of a few parameters for aural tuning on the whole piano tessitura. The parameters can be learned from the estimation of the inharmonicity coefficient and the fundamental frequency of a few single note recordings. For the inharmonicity coefficient, 3 notes in the bass range are sufficient to obtain a good interpolation on the whole tessitura. For the fundamental frequency, a few more notes are needed on the whole tessitura. The model takes into account physical considerations of the piano string scale design and piano tuning rules used by tuners.

It is intended to be useful for controlling the tuning of a given piano (as shown in Subsection 4.4) or for parametrizing the tuning of physically-based piano synthesizers. Following the steps proposed in this paper it could be possible to generate an inharmonicity curve specific to a given piano (or to set the inharmonicity coefficient of each note if the design of the target piano is perfectly known), choose the amount of stretching on the whole tessitura and an eventual global detuning to automatically generate an appropriate tuning.

The next step of this work is to include this model as constraints in multipitch (such as [17]) or automatic transcription algorithms of piano music. Instead of searching for 88 independent values of the inharmonicity coefficient and of the fundamental frequency, it strongly constraints the estimation to only 6 parameters, which should result in increased robustness.

6. ACKNOWLEDGMENTS

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A. APPENDIX: (F_0,B) ESTIMATION ON SINGLE NOTES

The problem of estimating (F_0,B) on single piano note recordings has been dealt with by several authors, amongst these: [13], [14], [15], [4]. Each of these algorithms could potentially be used for the estimation of the reference data used by the tuning model presented in the body of the article. However, the algorithm below comprises a new preprocessing stage of adaptive noise level estimation, which avoids most of potential outliers during the partial selection. Satisfactory results are usually obtained up to the C6 note.

Algorithm: The main idea is to perform a linear regression on an alternative version of the inharmonicity law (1):

$$\frac{f_n^2}{n^2} = F_0^2 + F_0^2 B \cdot n^2 \tag{18}$$

This equation is linear according to n^2. If we collect the f_n frequencies in the spectrum $S(f)$ and we know their rank n, we just have to do a linear regression to obtain F_0 and B. We use Least Absolute Deviation Regression (LADR) to discard outliers (phantom partials or partials affected by strong bridge coupling inharmonicity). The main steps of the algorithm are presented on Figure 6. The input is the magnitude spectrum $S(f)$ computed with zero padding on 2^{16} frequency bins from a 500ms window in the decay part of the sound. The first step is a noise level NL(f) estimation of the magnitude spectrum. This preprocessing stage allows the separation of spectral peaks related to partials from noise. Then, the partials above the noise level corresponding to transverse

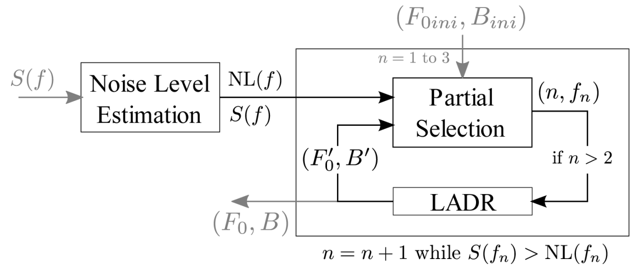

Figure 6: *Estimation of (F_0,B). Algorithm scheme.*

modes of vibration are picked up by an iterative process, estimating intermediate (F_0, B) values at each step.

Noise level estimation: We assume that the noise is an additive colored noise, i.e. generated by a filtered white gaussian noise [16]. In a given narrow band, if the filters have a quasi flat frequency response the noise can be considered as white gaussian, and its spectral magnitude follows a Rayleigh distribution:

$$p_X(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)} \tag{19}$$

In this pre-processing stage, we want to estimate the noise distribution in each band without removing the partials. To do so, a good estimator for σ is the median $med = \sigma\sqrt{\ln(4)}$. Indeed, in a given narrow band there are much less bins corresponding to partials than bins corresponding to noise, so partials have a reduced influence in the estimate of the noise median. The tradeoff sits in the choice of the bandwidth: the bands have to be narrow enough so that the white noise approximation holds, but wide enough so that most of the bins correspond to noise. We chose a 300Hz median filtering on the magnitude spectrum $S(f)$ to estimate $\sigma(f)$. Finally, we define the noise level in each band NL(f) as the magnitude such that the cumulative distribution function is equal to a given threshold T, set to $T = 0.9999$. With this choice of T, only 6 bins corresponding to noise on average (out of 2^{16}) should be above the noise level. The cumulative density function of a Rayleigh distribution is given by:

$$c_X(x;\sigma) = 1 - e^{-x^2/(2\sigma^2)} \tag{20}$$

Partial selection: The f_n are extracted in the same time as their rank n by an iterative process. We begin with an approximative value of $F_0 = F_{0\text{ini}}$ given by equal temperament (the processed note is supposed to be known) and with $B_{\text{ini}} = 0$, for the search of the first three partials. Then, for each iteration we perform LADR according to equation (18) to estimate an intermediate (F'_0, B') couple which will help in selecting the next partial. Each f_n frequency partial is searched in the range $nF'_0\sqrt{1 + B'n^2} + [-\frac{F_{0\text{ini}}}{5}, \frac{F_{0\text{ini}}}{5}]$. The width of the search interval is set empirically. Once no partial is found above the noise level the algorithm terminates. The last iteration is presented on Figure 7.

Influence of the dynamics: In practice, for a given note, sound spectra can significantly vary according to dynamics. For *forte* dynamics, a lot of “phantom” partials can appear in the spectrum (non-linear coupling of transverse waves with longitudinal waves), be picked during the partial selection and corrupt the linear regression. Another limitation can appear in *piano* dynamics: for sharp notes (from C6 to C8) the transverse mode partials are too weak and not in sufficient number to correctly process the selection and the regression steps.

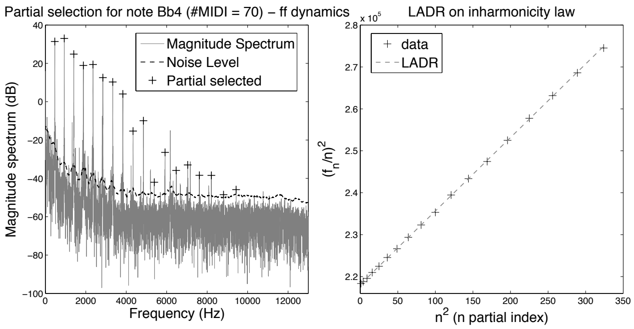

Figure 7: *Partial selection and LADR at the last iteration of the algorithm.*

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