VECTOR PHASESHAPING SYNTHESIS

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ABSTRACT

This paper introduces the Vector Phaseshaping (VPS) synthesis technique, which extends the classic Phase Distortion method by providing flexible means to distort the phase of a sinusoidal oscillator. This is achieved by describing the phase distortion function using one or more breakpoint vectors, which are then manipulated in two dimensions to produce waveshape modulation at control and audio rates. The synthesis parameters and their effects are explained, and the spectral description of the method is derived. Certain synthesis parameter combinations result in audible aliasing, which can be reduced with a novel aliasing suppression algorithm described in the paper. The extension is capable of producing a variety of interesting harmonic and inharmonic spectra, including for instance, formant peaks, while the two-dimensional form of the control parameters is expressive and is well suited for interactive applications.

1. INTRODUCTION

Abstract sound synthesis techniques have had a long history of development. Since the introduction of digital waveshaping in Risset's catalogue of computer instruments [1] in the late 1960s, subsequent theories related to non-linear distortion synthesis methods, such as FM, Discrete Summation Formulae (DSF) and others [2] [3] [4] [5] [6] [7] have emerged. Recently, the area has been revitalised with work on adaptive techniques [8] [9] [10], as well as new non-linear [11] and audio feedback methods [12] [13].

In this paper, we will start from an established non-linear Phase Distortion (PD) synthesis method [14], and propose three extensions to it in order to distort a sinusoidal waveform in a more complex manner: the inflection point is described as a two-dimensional vector, the phase distortion function is defined with multiple inflection points, and the modulation rate of the inflection points is raised to audio frequencies. These extensions allow a wider sonic palette to be extracted from the method, as well as more flexible control over the spectral changes. The new technique is named *Vector Phaseshaping* (VPS) synthesis.

Phaseshaping [15] [16] can be understood as a generalisation of the idea of PD, which in turn can be seen as a type of complex-wave phase modulation [11]. Phaseshaping is also related to non-linear waveshaping [17], but has some advantages over it. One of them is that in phaseshaping the use of non-smooth shaping functions does not necessarily imply the presence of audible aliasing, which is more or less inevitable in waveshaping. In addition, it is possible to mitigate the effects of aliasing, as will be explored later in this paper. Finally, waveshaping – as it is based on a non-linear amplification effect – requires care in terms of gain scaling to be usable. This is not, in general, a requirement for phaseshaping.

After a brief introduction to the original PD synthesis technique in Section 2, this paper is organized as follows. Section 3 introduces the VPS method, defines its multi-point vectorial extension, derives its spectral description, and proposes a novel alias-suppression method. Section 4 explores control rate modulation of the vector in 1-D, 2-D, and multi-vector configurations, while Section 5 complements this at audio rates. Finally, Section 6 concludes.

2. PHASE DISTORTION SYNTHESIS

The classic PD synthesis technique is defined by equations

$$s(n) = -\cos\{2\pi\phi_{\mathrm{pd}}[\phi(n)]\}, \tag{1}$$

$$\phi_{\mathrm{pd}}(x) = \begin{cases} \frac{1}{2}\frac{x}{d}, & 0 \leq x \leq d \\ \frac{1}{2}[1 + \frac{(x-d)}{(1-d)}], & d < x < 1, \end{cases} \tag{2}$$

where n is the sample number and d is the point of inflection (see Fig. 1). In this case, equation (2) is a phaseshaper acting on an input signal $\phi(n)$. This is a trivial sawtooth wave with frequency f_0 and sampling rate f_{s}, and given by

$$\phi(n) = [\frac{f_0}{f_s} + \phi(n-1)] \bmod 1, \tag{3}$$

which is same as the phase signal used in a standard table lookup oscillator, for instance. The mod 1 operator can be defined as

$$x \bmod 1 \triangleq x - \lfloor x \rfloor, x \in \mathbb{R}. \tag{4}$$

In this particular phaseshaper, the point of inflection d determines the brightness, the number of harmonics, and therefore the shape of the output signal. The closer d is to 0 or to 1, the brighter the signal (and more prone to audible aliasing). At $d = 0.5$, there is no change in the phase signal as the shaper function is linear. In fact, there is a symmetry condition around $d = 0.5$, with the output being based on a falling shape with $0 \leq d < 0.5$ and a rising shape with $0.5 < d \leq 1$. By varying d, we can get an effect that is similar to changing the cutoff frequency of a low-pass filter. The PD equation can also be cast as a case of complex-wave phase modulation, as discussed in [11] and [15].

3. VECTOR PHASESHAPING SYNTHESIS

This section introduces the VPS method, which is a new extension to the phase distortion synthesis technique described above. In classic PD, the inflection point d controls the x-axis position of the phase distortion function bending point, which traces the thin

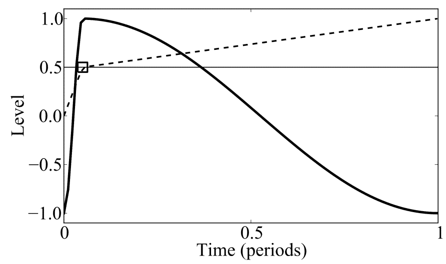

Figure 1: *Classic PD sawtooth waveform (thick), phaseshaping function (dashed), and inflection point path (thin horizontal line). The inflection point d = 0.05 is marked with a square.*

horizontal path shown in Fig. 1. Thus, the vertical position of the bend is fixed at 0.5. VPS synthesis releases this constraint and expresses the inflection point as a two-dimensional vector

$$p = (d, v), \quad (5)$$

where d is the horizontal, $0 \leq d \leq 1$, and v is the vertical position of the bending point. The two-dimensional phase distortion function is given by

$$\phi_{\text{vps}}(x) = \begin{cases} \frac{vx}{d}, & 0 \leq x \leq d \\ (1-v)\frac{(x-d)}{(1-d)} + v, & d < x < 1, \end{cases} \quad (6)$$

which reduces to the classic form of equation (2) when $v = 0.5$.

To gain an understanding of the effect of v, consider first setting $p = (0.5, 0.5)$, which, when applied to the waveshaper of equation (1), produces an undistorted inverted cosine waveform as shown in Fig. 2(a). As v is then raised towards unity, the slope of the first segment of equation (6) is increased as well, and the waveshaped output of the first segment grows towards a full-cycle sinusoid (Fig. 2(b), before period 0.5). Consequently, the slope of the second segment becomes less steep, and at $v = 1$, it produces the static portion of the waveform depicted in Fig. 2(c).

The bandwidth of the spectrum grows from a single harmonic to the form shown in Fig. 2(b), and then shrinks towards that of Fig. 2(c). Since the latter waveform resembles a half-wave rectified sinusoid, its spectrum consists of odd harmonics, with a strong additional second harmonic.

On the other hand, if v is decreased from 0.5 towards 0, the slope of the first segment decreases while that of the second one increases. The waveforms of Fig. 2 become then reversed in time, and therefore, identical spectra are obtained for v values that are symmetric around 0.5. Since both vertical and horizontal domains of the inflection vector are symmetric around this position, the point p_0 = (0.5,0.5) defines the center location of the two-dimensional VPS parameter space. However, when either d or v is offset from its center position, this symmetry property (of the other parameter) is no longer sustained. This leads to some interesting characteristics that will be discussed next.

When $v = 1$, as in Fig. 2(c), d controls the duty width of the produced waveform. The pulse width increases with d, from a narrow impulse up to a full-cycle sinusoid at $d = 1$. Pulses of various widths can then be constructed with $p = (d, 1)$ and $0 < d \leq 0.5$, or $p = (d, 0)$ and $0.5 \leq d < 1$ (these are symmetrically-related vectors), as seen in Fig. 3. Transitions between various pulse widths and other waveshapes can be smoothly created by interpolating the vector values. These effects will be particularly interesting when vectors are subjected to modulation. Classic oscillator effects such as pulse-width modulation will be easily implemented by modulating the d value of the vector with a low-frequency oscillator (LFO).

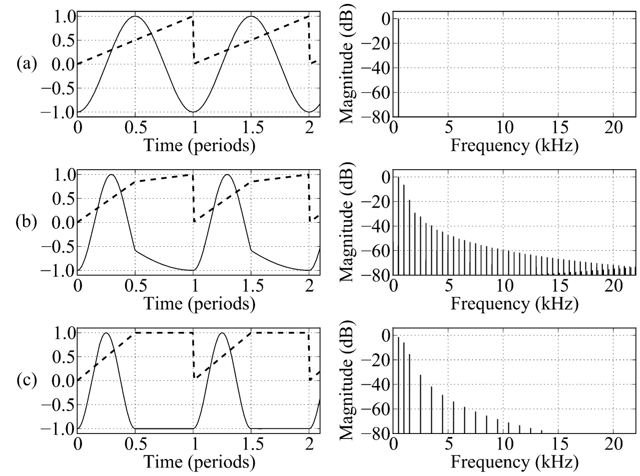

Figure 2: *VPS waveforms and spectra of (a) p=(0.5,0.5), (b) p=(0.5,0.85), and (c) p=(0.5,1).* $f_0 = 500$ *Hz and* $f_s = 44100$ *Hz, as in all examples of this paper.*

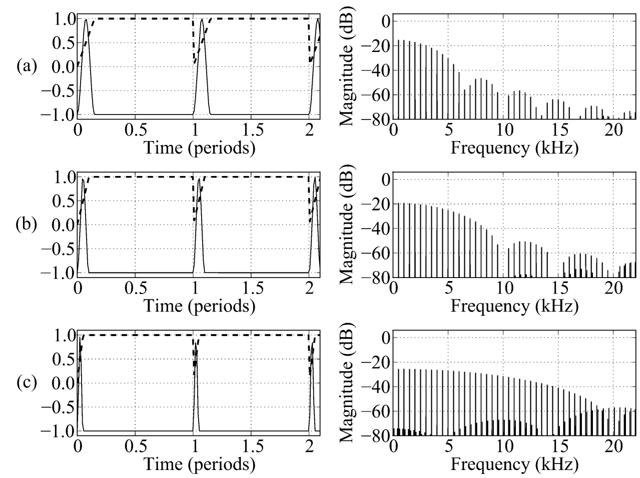

Figure 3: *VPS pulse waveforms and spectra of (a) p=(0.15,1), (b) p=(0.1,1), and (c) p=(0.05,1).*

Certain combinations of v and d will always produce sinusoids. For instance, $v = 1, 2, 3...n$ and $d = 1$ forms a single-segment linear phaseshaper, and similarly, $v = d = 0.5$ (i.e., the trivial form) is linear, but there are other cases. For instance, sinusoids will be produced with $v = 1.5$ and $d = 0.75$, and with $v = 3$ and $d = 0.6$. The general form of this, for $0 < d < 1$, is

$$|\frac{v}{d}| = |\frac{1-v}{1-d}|, v/d \in \mathbb{Z}. \quad (7)$$

This is because the derivative of the phase on both sides of the inflection point has the same absolute value (it might only differ in sign). Due to the use of a cosine wave, which is an even function, the change of sign of the derivative at the inflection point happens to be of no consequence. The pitch of the produced sinusoid will be equivalent to $|v/d|f_0$, the fundamental frequency times the absolute value of the phase derivative. The presence of these sinusoid cases means that, when changing the waveshapes by manipulating the vector, there will be loci of p where all components suddenly vanish, leaving a single harmonic sounding. This effect can be quite dramatic and of musical interest.

3.1. Synthesising Formants

At $d = 0.5$ and with $v \geq 1.5$ VPS produces formants, which are quite prominent when centred around exact harmonics of the fundamental. In such situations, the spectrum consists of five harmonics around a central frequency and of sidebands of odd order harmonics, as shown in Fig. 4(a). As can be seen, the magnitude of the emphasised frequency region is strong in comparison to the sidebands, which is a useful property in vocal and resonant filter synthesis applications.

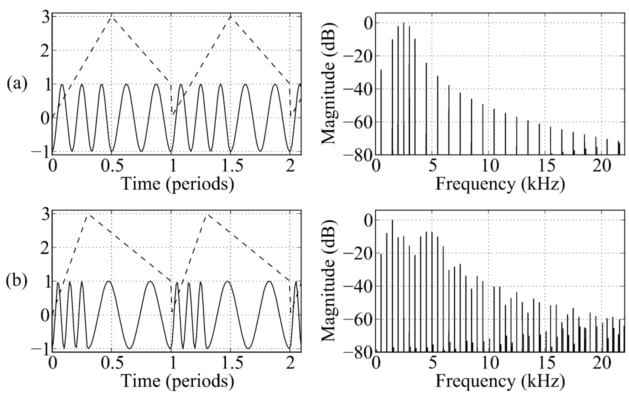

Figure 4: *(a) VPS formant, $p = (0.5, 3)$, and (b) multiple formants ($d \neq 0.5$).*

In these cases, the ratio of the formant centre frequency f_{f} and the fundamental frequency f_0 is defined as

$$\frac{f_{\mathrm{f}}}{f_0} = 2v - 1, \tag{8}$$

so formants will be centred on exact harmonics of f_0 when $v = 1.5, 2, 2.5, ...$

Offsetting d from its central position spreads the formant across the digital baseband as shown in Fig. 4(b). As in classic PD, d controls the spectral brightness of the timbre, and as it approaches 0 or 1, there will be an increased amount of aliasing in the output spectra. This is due to the high-frequency periods in the shorter segment. Aliasing can also happen when equation (8) does not yield integral values. The reason now is that there will be incomplete periods, i.e., discontinuities in the waveform itself or in its first derivative.

One possible way to counteract this last effect has been demonstrated in other techniques, such as PAF [7] and phase-synchronous ModFM [18]. These techniques also share a similar problem whereby if the formant frequency is not an exact multiple of the fundamental, discontinuities in the output waveform can occur. The solution is to use two oscillators, whose formant frequencies are tuned to adjacent multiples of the fundamental around the exact formant frequency that is required. A simple linear crossfading of the two output signals will generate a peak at the target formant centre. This allows us to sweep the spectrum smoothly with no aliasing noise due to this particular effect. To achieve this, we define an interpolation gain a that is dependent on the fractional part of $f_{\mathrm{f}} : f_0$,

$$a = [2v - 1] \bmod 1, \tag{9}$$

and then use it to scale the two VPS signals, $s_1(n)$ and $s_2(n)$, employing inflection vectors $p_1 = (0.5, v)$ and $p_2 = (0.5, v + 0.5)$, respectively, with $v > 1$ and $2v - 1 \in \mathbb{Z}$ to obtain the output signal $y(n)$:

$$y(n) = (1 - a)s_1(n) + as_2(n). \tag{10}$$

3.2. Aliasing Suppression

The aliasing produced by the incomplete periods may also be suppressed by exploiting a novel single oscillator algorithm, which modifies the phaseshaper when $\phi_{\mathrm{vps}}[\phi(n)] > \mathrm{floor}(v)$, i.e., when the phase is inside the incomplete period. The modified phaseshaper is given by

$$\phi_{\mathrm{a}}(x) = \begin{cases} \frac{p \bmod 1}{2b}, & 0 < b \leq 0.5 \\ \frac{p \bmod 1}{b}, & 0.5 < b < 1, \end{cases} \tag{11}$$

where $p = \phi_{\mathrm{vps}}(x)$ and $b = v \bmod 1$. When applied to equation (1), the incomplete segment is rendered as a smooth full-cycle sinusoid, which is then scaled and offset in relation to $c = \cos(2\pi b)$:

$$s_{\mathrm{a}}(n) = \begin{cases} [(1 - c)s(n) - 1 - c]/2, & 0 < b \leq 0.5 \\ [(1 + c)s(n) + 1 - c]/2, & b > 0.5, \phi_{\mathrm{a}}(x) > 0.5, \end{cases} \tag{12}$$

Fig. 5 shows that the aliasing present in the trivial VPS form (Fig. 5(a)) is reduced substantially when processed with this algorithm (Fig. 5(b)). This is achieved at the cost of reduced high-end spectral content.

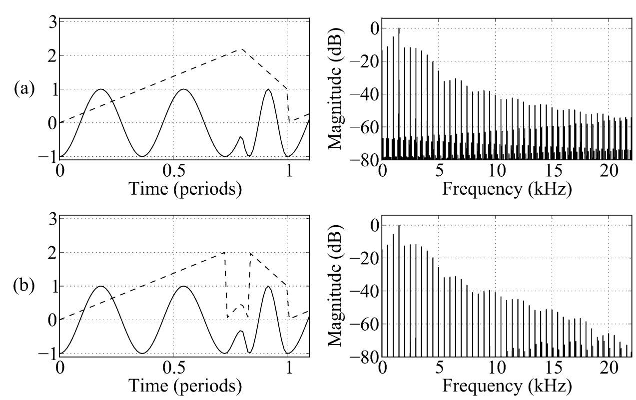

Figure 5: *(a) Trivial VPS timbre and (b) its alias-suppressed form. $p = (0.8, 2.2)$.*

3.3. Notes on the Derivation of the VPS Spectrum

Given that VPS, as an extension of PD, is effectively a complex form of Phase Modulation (PM) synthesis, it is reasonable to expect that we would be able to derive a closed-form expression for its spectrum. In order to do this, we would put the technique in terms of a carrier phase, to which a given modulation function is applied. In [11] this is implemented for the PD modulation function of equation (2) using a complex modulating wave PM spectrum derivation [19]. This yields an expression with many terms based on a product series of Bessel coefficients that is very unwieldy and difficult to handle.

For simple geometric PM functions (such as the ones found in VPS), it is, however, possible to obtain an alternative spectral description that avoids the use of Bessel coefficients. This derivation method is similar to the one in [20]. For a VPS function with v and d, we can have a corresponding PM function $M(x) = \pi d\{2[\phi_{\mathrm{vps}}(x + dx) - x - dx] - 1\}, -\pi \leq x \leq \pi$, such that,

$$-\cos\{2\pi\phi_{\mathrm{vps}}[\phi(t)]\} = -\cos\{\omega t + M(t) + \phi_d\} = \\ \sin(\omega t + \phi_d)\sin[M(t)] - \cos(\omega t + \phi_d)\cos[M(t)], \quad (13)$$

$$\cos[M(t)] = \begin{cases} \cos(y\theta), & 0 \leq \theta \leq \pi x \\ \cos[\frac{xy(\pi-\theta)}{1-x}], & \pi x < \theta \leq \pi, \end{cases} \quad (14)$$
$$\cos[M(t)] = \cos[-M(t)],$$

and

$$\sin[M(t)] = \begin{cases} \sin(y\theta), & 0 \leq \theta \leq \pi x \\ \sin[\frac{xy(\pi-\theta)}{1-x}], & \pi x < \theta \leq \pi, \end{cases} \quad (15)$$
$$\sin[M(t)] = -\sin[-M(t)],$$

where t is time, $\phi_d = 2\pi d, x = d/2$, and $y = v(1 - d)/2$.

Because $\cos[M(t)]$ and $\sin[M(t)]$ are even and odd, respectively, in order to compute their Fourier series, we only need half periods, as defined by equations (14) and (15). Using these series and the right-hand side of equation (13), we can obtain a description of the VPS spectrum as

$$s(t) = -b_0'\cos(\omega t + \phi_d) + \sum_{n=1}^{\infty} \frac{c_n - b_n}{2}\cos(\omega t[n-1] + \phi_d) - \\ \frac{c_n + b_n}{2}\cos(\omega t[1+n] + \phi_d) \quad (16)$$

with

$$b_n = \frac{2}{\pi}\int_0^{\pi x}\cos(y\theta)\cos(n\theta)d\theta + \\ \frac{2}{\pi}\int_{\pi x}^{\pi}\cos(\frac{yx(\pi-\theta)}{1-x})\sin(n\theta)d\theta, \quad (17)$$

$$c_n = \frac{2}{\pi}\int_0^{\pi x}\sin(y\theta)sin(n\theta)d\theta + \\ \frac{2}{\pi}\int_{\pi x}^{\pi}\sin(\frac{yx(\pi-\theta)}{1-x})\sin(n\theta)d\theta, \quad (18)$$

and $b_0' = \frac{b_0}{2}$.

With these expressions, it is possible to derive the spectrum of single inflection point VPS. However, in our studies, we have found that the VPS method is at times more simply described in terms of waveform morphologies linked to the geometry of phaseshaping functions. Some of these are described in Table 1.

3.4. Multiple Inflection Points

Finally, it is interesting to consider the possibility of more than one inflection points. This can be used to obtain, for instance, square wave-like output signals. Consider for instance the use of three vectors $p_0 = (d_0, v_0)$, $p_1 = (d_1, v_1)$, and $p_2 = (d_2, v_2)$. With $p_0 = (0.1, 0.5)$, $p_1 = (0.5, 0.5)$, and $p_2 = (0.6, 1)$, we have a wave that approximates a square shape (see Fig. 6). These three points, however, can be freely manipulated to provide a variety of waveforms.

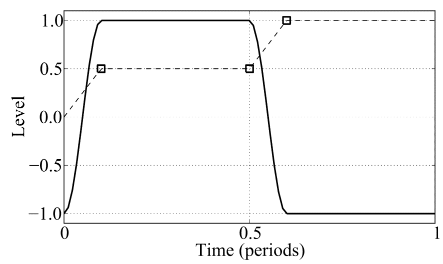

Figure 6: *Square-like waveform (thick) produced by a phase-shaping function (dashed) with three inflection points at* $p_0 = (0.1, 0.5)$*,* $p_1 = (0.5, 0.5)$*, and* $p_2 = (0.6, 1)$.

The general form for the phaseshaping function in Multiple Vector Phaseshaping with N inflection points $p_0, p_1, ...,$ p_{N-1} is:

$$\phi_{\mathrm{mvps}}(x) = \begin{cases} \frac{v_0 x}{d_0}, & x < d_0 \\ (v_n - v_{n-1})\frac{(x-d_{n-1})}{(d_n-d_{n-1})} + v_{n-1}, & d_{n-1} \leq x < d_n \\ ... \\ (1 - v_{N-1})\frac{(x-d_{N-1})}{(1-d_{N-1})} + v_{N-1}, & x \geq d_{N-1} \end{cases} \quad (19)$$

which reduces to equation (6) for $N = 1$.

4. ADAPTIVE VECTOR CONTROL

VPS synthesis provides rich spectra of various forms. In order to seize the musical potential from the method, we need to provide adaptive controls [21] to the vector parameters. In this section, we will examine various possibilities arising from this.

4.1. One-dimensional Control

The simplest means of adaptive control over waveform shapes is provided by varying one of the vector parameters, whilst holding the other constant. For instance, keeping $v = 0.5$ and varying d provides an emulation of a low-pass filter sweep, as the spectrum gets richer with d close to 0 or 1. Keeping $d = 0.5$ and varying v also provides a similar effect, but now with a resonant peak at $(2v - 1)f_0$, as discussed earlier. Other fixed values of d and v will create transitions between the various characteristics outlined in the previous section.

4.2. Two-dimensional Low-frequency Modulation

It is with two-dimensional adaptive control, however, that VPS synthesis becomes a very original proposition. This can be performed by a joystick or an *x-y* controller. Transitions between a

Table 1: *Some VPS morphologies.*

$p(d, v)$	**Waveshape**	**Spectrum**	**Figure**
$(0.5, 1)$	half sinusoid	missing some even harmonics	2(c)
$(0.5, < 1)$	distorted half sinusoid	steep spectral slope	2(b)
$(0.5, > 1)$	varying-period sinusoids	peaks at $(2v - 1)f_0$	4(a)
$(< 0.5, 1)$	pulse-like	spectrum gets richer with $d \to 0$	3
$(< 0.5, > 1)$	varying-period distorted sinusoids	multiple formant peaks	4(b)
$(< 0.5, 0.5)$	sawtooth-like	more gradual spectral slope	1

variety of waveshapes (e.g., with those summarised in Table 1) can be easily achieved, and this can provide a great source of timbral expression in musical performance.

This facility can be extended by the use of a 2-D LFO. This can take the form of two separate oscillators, controlling the two parameters d and v, i.e., the rectangular coordinates of the vector, or its polar representation, the angle and magnitude. The oscillators can exhibit different waveshapes and frequencies.

4.3. Lissajous Modulation

An interesting application of a 2-D LFO can be achieved by synchronising the phase of the two oscillators, so that the path of the modulated inflection point forms a *Lissajous figure* [22]. This is achieved by combining the two modulator signals in the following system of equations:

$$\begin{cases} d = A_d\{0.5 + 0.5\cos(\omega_d + \theta)\} \\ v = A_v\{0.5 + 0.5\cos(\omega_v)\}, \end{cases} \tag{20}$$

with $0 \leq A_d \leq 1$ and $A_v \geq 0$.

Various interesting 2-D modulation shapes can then be obtained. With $\omega_d = \omega_v$ and $\theta = \pi/2$ we can create circular or elliptical paths. If the two LFO frequencies are different, $\omega_d = n\omega_v$ or $\omega_v = m\omega_d$ and $\theta = \pi/2$, we will have n vertical or m horizontal 'rings' ($m, n \in Z$ and $m > 1$). By varying θ, we can also collapse the path into a straight diagonal line ($\omega_d = \omega_v$ and $\theta = 0$, ascending; or $\theta = 0.5$, descending). Complex paths can be created by varying these parameters. Fig. 7 shows various combinations of these Lissajous modulation paths, while Fig. 8 shows a spectrogram of a Lissajous-modulated VPS timbre. In addition, a second-order modulator can be employed to control these parameters for a cyclical modulation path transformation.

4.4. Multi-vector Modulation

Finally, we must consider the possibilities of multi-vector modulation. Here, the multiplication of parameters might pose a problem for controller mapping. In addition, the horizontal component of each vector will work on a limited range, which will depend on the positions of neighboring inflection points. This is required so that the phaseshaping function remains single-valued. However, bounds for the vertical component work as before.

One solution to this issue is to use a single controller (such as a 2-D LFO or a joystick), which would determine the positions of all inflection points by a mapping matrix. The advantage of this is that principles developed for single vectors, such as Lissajous modulation, can be easily extended to this case.

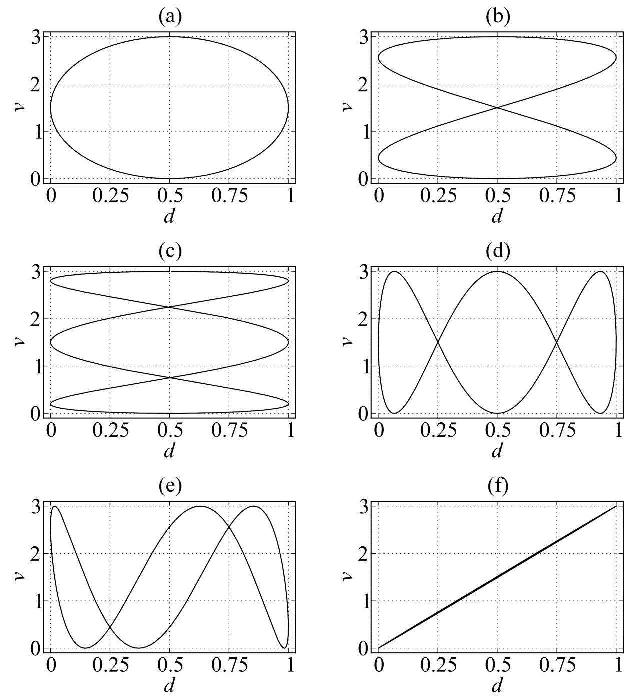

Figure 7: *Lissajous modulation, with $A_d = 1$ and $A_v = 3$: (a) $\omega_d = \omega_v$ and $\theta = \pi/2$; (b) $\omega_d = 2\omega_v$ and $\theta = \pi/2$; (c) $\omega_d = 3\omega_v$ and $\theta = \pi/2$; (d) $\omega_v = 3\omega_d$ and $\theta = \pi/2$; (e) $\omega_v = 3\omega_d$ and $\theta = \pi/4$; (f) $\omega_d = \omega_v$ and $\theta = 0.01$.*

To explore independent modulation of vectors, a solution can be found in multi-touch controllers, where each inflection point can be determined by finger position. Given that each segment of the phaseshaping function will be independent, this type of adaptive control can be used to create wave sequencing effects.

5. AUDIO-RATE WAVESHAPE MODULATION

Other types of complex spectra are obtained by extending the application of modulation to audio frequencies. As in the previous section, we will first look at the modulation of individual vector components, then study the combination of the two dimensions.

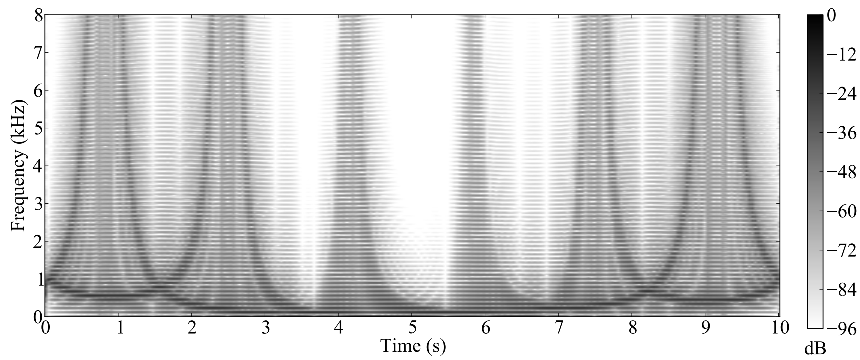

Figure 8: *Spectrogram of VPS with Lissajous modulation,* $\omega_d = 3\omega_v$*,* $\theta = \pi/2$*,* $A_d = 1$ *and* $A_v = 5$*.*

5.1. One-dimensional Vector Modulation

Starting with the original PD arrangement, with $v = 0.5$, and modulating d, we observe two basic types of output for $f_{\rm m} = f_0$ (with $f_{\rm m}$ as the modulator frequency):

1. Single-sided modulation, where $0 \leq d \leq 0.5$ or $0.5 \leq d \leq 1$, using an inverted cosine modulator produces a spectrum that decays more abruptly than $1/f$ and a sawtooth-like waveshape, see Fig. 9(a). The bigger the phase difference between the modulator and an inverted cosine, means the brighter the spectrum. The spectrum is brightest overall with a cosine modulator.
2. Double-sided modulation, where $0 \leq d \leq 1$, we observe a peak at the second harmonic, and a more gradual decay in the spectral envelope, with a cosine or inverted cosine modulator. Discontinuities in the waveform reset also produce substantial aliasing, see Fig. 9 (b). The modulator phase also has an effect: high frequency components will be substantially attenuated and the peak at the second harmonic disappears when the modulator is a sine wave, see Fig. 9(c).

By reducing the modulation amount, less components will be generated, so this parameter can be used as a timbre control. In general, a fundamental difference between static or low-frequency modulated and audio-rate modulated VPS is that in the latter case, the phaseshaping functions are not based on linear segments, as seen in Fig. 9.

On the other hand, if we hold d static at 0.5 and vary v, sinusoidally, $f_{\rm m} = f_0$ and $0 \leq v \leq k$, we will have a bright spectrum that depends on the modulation width k. Higher values of k will distribute the energy more evenly and produce a richer spectrum. At the highest values of k, spectral peaks will be less pronounced. Interestingly, the output will also be quasi-bandlimited, so it is possible to suppress aliasing by keeping k under control. Modulator phase will also affect the output waveform shape and spectrum. Fig. 10 shows three different cases of this type of modulation.

Finally, we must consider the cases where $f_{\rm m} \neq f_0$. When $f_{\rm m} = nf_0, n \in \mathbb{Z}$, the spectrum will be harmonic and generally increasing in brightness with n. If $f_m << f_0$, the perceived fundamental will not be equivalent to f_0 anymore. In cases where $f_{\rm m} = f_0/n, n \in \mathbb{Z}$, we will have a harmonic spectrum with $f_{\rm m}$ as the fundamental. When $f_{\rm m}/f_0$ is not a ratio of small numbers or is irrational, we will have an inharmonic spectrum. The reason for this is that the output will be composed of a fast sequence of different waveshapes, which will not be fused into a periodic pattern. Interesting cases happen when $f_{\rm m}$ and f_0 are very close, but not exactly the same value. In these cases, the spectrum will be harmonic and we will perceive a cyclically changing timbral pattern whose period is $1/(f_{\rm m} - f_0)$. The spectrogram of such a tone is shown in Fig. 11, where $f_{\rm m} = f_{\rm c} - 0.1$. These observations regarding the modulation frequency are similarly applied to the two-dimensional cases discussed below.

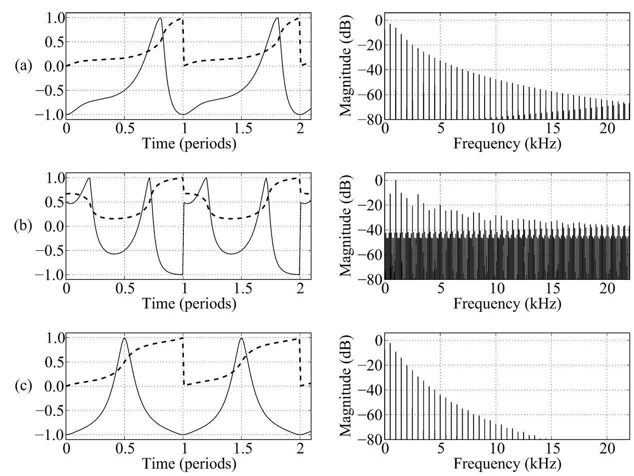

Figure 9: *Audio-rate shape modulation of vector component* d*,* $f_{\rm m} = f_0$ *(output, solid line; phase, dashed line): (a) Single-sided, inverted cosine modulator, (b) double-sided, inverted cosine modulator, and (c) double-sided, sine modulator.*

5.2. Two-dimensional Vector Modulation

Two-dimensional audio-rate modulation is more conveniently implemented using the Lissajous arrangement introduced in the previous section. In the case of audio-rate modulation, its parameters will be modulation frequency f_m, horizontal to vertical frequency ratio $\omega_d : \omega_v$ (which scales the modulation frequency for each component), horizontal phase difference θ, and horizontal and vertical modulation width, A_d and A_v, respectively. Fig. 12 shows three examples of different timbres produced by varying the Lissajous parameters, all of them with $f_m = f_c$, defining three dif-

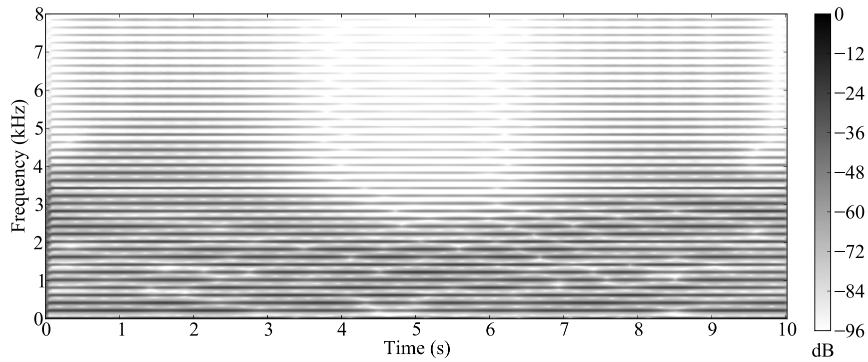

Figure 11: *Spectrogram of an audio-rate modulated VPS timbre showing a cyclically changing spectrum, $d = 0.5$ with v modulated by an inverted cosine, $k = 5$, $f_{\rm c} = 500$ Hz, and $f_{\rm m} = f_{\rm c} - 0.1$.*

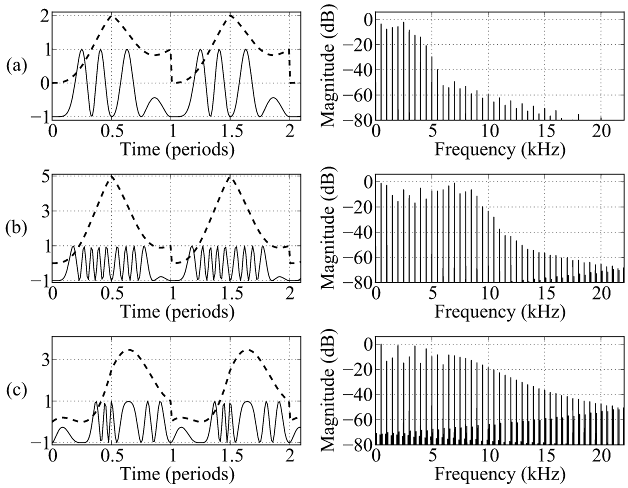

Figure 10: *Audio-rate shape modulation of vector component v, $f_{\rm m} = f_0$ (output, solid line; phase, dashed line): (a) modulator width $k = 2$, (b) $k = 5$ (cosine phase) and (c) $k = 2$ (sine phase).*

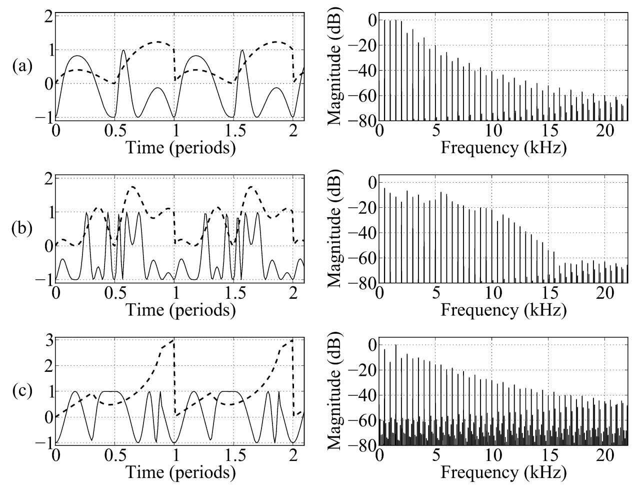

Figure 12: *Lissajous audio-rate shape modulation of vector component v, $f_m = f_0, A_d = 1$ and $A_v = 3$ (output, solid line; phase, dashed line): (a) $\omega_d = \omega_v$ and $\theta = -\pi/2$ (circular path); (b) $\omega_v = 3\omega_d$ and $\theta = -\pi/2$ (three vertical rings); (c) $\omega_d = \omega_v$ and $\theta = 0.01$ (nearly linear path).*

ferent modulation paths: circular, three vertical rings and nearly linear.

The advantages to the use of Lissajous modulation is that it is possible to identify, in general lines, a particular modulation path with a given tone spectrum. Therefore, the principles of defining morphologies, which was done in Table 1 for the VPS tones, can be extended to the more complex 2-D audio-rate shape-modulated timbres. This could be the basis for the selection of desired timbral qualities, with a simpler and more compact parameter set than the case of independent modulators for the two dimensions. For instance, it is clear from Fig. 12 that some paths will be more prone to aliasing (e.g., Fig. 12 (c)), whereas others provide a cleaner spectrum.

There is no doubt that 2-D audio-rate modulation will inevitably produce aliasing of some kind. How objectionable this might be is probably a better question. From a musical point of view, what we observe in the output is that some types of modulation will generate aliasing that is perceptually a form of bright broadband noise, and that is a sonority that is in some cases desirable. In this case, a technique for a synthesiser with wide musical applications might embrace the production of some amount of aliasing as one of its characteristics, rather than consider it to be a defect.

5.3. Multi-vector Manipulation

Completing the types of audio-rate shape modulation that are possible with VPS, we have multi-vector manipulation. Here, at least four scenarios can be described:

- One-dimensional manipulation of a single vector (other vectors constant)
- One-dimensional manipulation of multiple vectors.
- Two-dimensional manipulation of a single vector.
- Two-dimensional manipulation of multiple vectors.

Regarding these different cases, a few general lines can be discerned. The first case of one-dimensional modulation is effectively an extension of the single-vector case, and it is possible to apply the principles discussed above to it. Similarly, two-dimensional manipulation can be seen as an extension of ideas previously discussed in this paper, both in low-frequency modulation of multiple vectors and in audio-rate Lissajous modulation.

The most complex case to handle, however, is possibly one-dimensional manipulation of multiple vectors. Various sub-cases can arise from this, since each inflection point can be modulated in either direction. It is possible to simplify our approach and use a single modulator that is mapped to different vectors/directions, as in the case of the matrix mapping discussed earlier on in Section 3.4. The ranges of the horizontal component of each point will have to be scaled properly so that no overlap occurs. In any case, multi-vector manipulation is by far the most complex case of VPS synthesis and it requires a detailed study that is left as future work.

6. CONCLUSIONS

This paper introduced the technique of Vector Phaseshaping synthesis as an extension of the well-known Phase Distortion method. Its main characteristics and spectral description were defined in detail. A novel alias-suppression method was also described, and the various methods of timbre modification via low-frequency modulation were discussed. In a complementary manner, we also looked at audio-rate shape modulation synthesis and discussed the general principles of the technique. It is expected that the ideas proposed in this paper will find a good range of applications in musical sound synthesis. Sound examples and software are available at `http://www.acoustics.hut.fi/go/dafx11-vps`.

7. ACKNOWLEDGMENTS

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