LOW FREQUENCY GROUP DELAY EQUALIZATION OF VENTED BOXES USING DIGITAL CORRECTION FILTERS

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ABSTRACT

In this paper methods to determine the group delay of vented boxes and techniques for the design of filters for group delay equalization are presented. First the transfer function and the related group delay are explained. Then it is shown how the group delay can be computed or approximated for a certain alignment of the box. Furthermore it is shown how to derive the required parameters of the transfer function from a simple electrical measurement of the box, which allows the determination of the group delay without knowledge of the box design parameters. Two strategies for the design and implementation of digital correction filters are shown where one approach allows for a real-time adjustability of the delay. Finally, the performance with a real speaker is evaluated.

1. INTRODUCTION

Vented boxes have been in use for a long time. Their theory was described the first time to a great extent by Thiele in [1] and [2]. Later Small refined the theory further [3]-[4]. Both authors provided a mathematical description of a vented loudspeaker that allowed for a systematic design and an assessment of the transfer characteristics, which was not the case before. Later on, Bullock [5] streamlined the design procedure and made the data provided by Thiele and Small more practically usable.

The advantage of vented boxes w.r.t. closed or dipole speakers is their enhanced bass response. Their drawback is an increased group delay at low frequencies, which among other effects, can lead to the perception of a "muddy", "boomy" or "slow" bass.

These deficiencies at low frequencies are not the only phase errors of loudspeakers. In general, modern speakers are multi-way systems and the multiple ways are separated by a crossover, which can be implemented as a passive, an active analogue or an active digital system. Ideally, the output of the paths add up to a constant frequency response. A crossover is made of filters which provide the desired frequency division, but also introduce unwanted phase shifts and hence group delay errors. Additionally, the placement of the speakers relative to each other can introduce time-alignment errors. The significance and audibility of these phase or group delay errors is subject to ongoing research and discussions, see [6], [7], [8], [9] for example. Time-alignment correction using group delay equalization is proposed by [10] and [11]. For example [12] and [13] propose the correction of phase distortion with allpass filters.

Most of the present work deals with higher frequencies and equalization in the low-frequency region is rarely discussed. Linkwitz [14] states that it is not a trivial task, since a lot of delay is needed at higher frequencies of the spectrum. The authors of [15] focus on the phase correction at higher frequencies and remind that the low frequency sound is not perceived independently of the characteristics of the listening room. This is of course true but not limited to the low frequency range and a loudspeaker as ideal as possible is desirable.

In this paper we will focus on the equalization of group delay errors that are introduced by the driver-enclosure-system in the low-frequency range.

2. FUNDAMENTALS OF VENTED BOXES

A vented box is a loudspeaker enclosure with an additional opening called a vent or port, which is usually made of a tube.

The behaviour of a vented box can be described as a fourth order highpass system. The box itself is a resonator with the air in the box volume acting as a spring and the air in the port behaving as a mass, which together are a resonating mass-spring-system. Such a system is also known as a Helmholtz resonator.

The resonator helps the chassis to reproduce low frequency bass but has the disadvantage that the onset and offset of its oscillation is somewhat delayed to the driver signal. At the box resonance frequency the sound output is nearly solely coming from the port and thus has the group delay of the resonator. For frequencies higher than the port resonance frequency, the sound output from the driver and the port are mixed and at higher frequencies the sound from the driver dominates. Fig.1 illustrates the typical behaviour of a vented box derived from a LTspice simulation [16]. The group delay in this example exhibits a maximum of about 18 ms slightly below the port resonance frequency of about 34 Hz.

2.1. Transfer function of a vented box

The sound pressure frequency response of a vented box can be expressed by the general system function of a fourth order highpass:

$$G_v(s) = \frac{\left(\frac{s}{\omega_0}\right)^4}{1 + a_1\left(\frac{s}{\omega_0}\right) + a_2\left(\frac{s}{\omega_0}\right)^2 + a_3\left(\frac{s}{\omega_0}\right)^3 + \left(\frac{s}{\omega_0}\right)^4} \tag{1}$$

There are also higher order systems possible that make use of additional electrical filters to shape the low-end frequency response. These assisted designs are not considered here, since they are quite unusual. The coefficients of the system function $G_v(s)$ are related to the design parameters of the vented speaker and are defined by

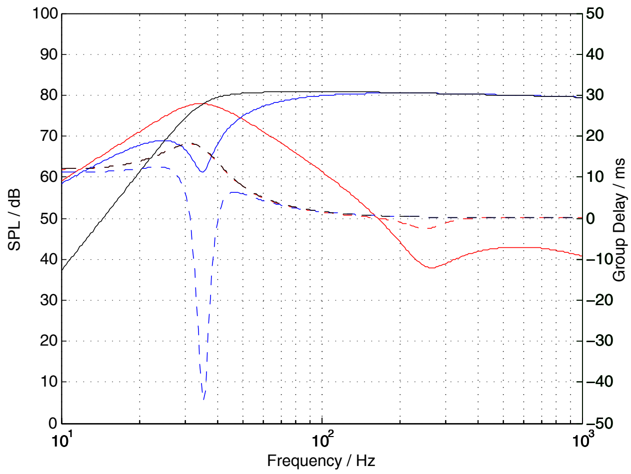

Figure 1: *Spice simulation of vented box showing magnitude (—) and group delay (- -) responses of the driver (blue), the port (red) and the combined output (black).*

the relationships

$$a_1 = \frac{1}{Q_l\sqrt{h}} + \frac{\sqrt{h}}{Q_{ts}} \quad (2)$$

$$a_2 = \frac{\alpha+1}{h} + h + \frac{1}{Q_l Q_{ts}} \quad (3)$$

and

$$a_3 = \frac{1}{Q_{ts}\sqrt{h}} + \frac{\sqrt{h}}{Q_l}. \quad (4)$$

$\alpha = V_{as}/V_B$ is the system compliance ratio. It describes the ratio of the compliance of the air in the box V_B to the compliance of the low-frequency driver V_{as}. $h = f_b/f_s$ is the tuning ratio, which is the ratio of the free-air resonance of the driver f_s to the resonance frequency of the box f_b. Both α and h are determined in the box design process to meet specific requirements. Q_l is the quality factor of the enclosure and depends on the construction of the box with regard to losses and air tightness. For a medium sized box with slight damping at the inner walls $Q_l = 7$ can be assumed. Q_{ts} is the total quality factor of the driver including mechanical and electrical characteristics and additionally resistive contributions from the crossover [17].

Depending on the values of the coefficients, the response of a box is classified as a certain alignment. The choice of an alignment depends on the desired frequency response and thus has an influence on group delay at low frequencies. Not all alignments are possible with all drivers depending on their parameters.

The alignment is usually derived from the magnitude squared function setting $s = j\omega$ and $\widehat{\omega} = \omega/\omega_0$, where ω_0 is the corner frequency of the highpass as

$$|G_v(j\omega)|^2 = \frac{\widehat{\omega}^8}{1 + A_1\,\widehat{\omega}^2 + A_2\,\widehat{\omega}^4 + A_3\,\widehat{\omega}^6 + \widehat{\omega}^8} \quad (5)$$

with

$$A_1 = a_1^2 - 2a_2\,, \quad A_2 = 2 + a_2^2 - 2a_1a_3\,, \quad A_3 = a_3^2 - 2a_2 \quad (6)$$

The box design parameters are then computed from A_1, A_2 and A_3.

2.2. B4 alignment (fourth order Butterworth)

For this alignment the transfer function corresponds to that of a fourth order Butterworth highpass. It is characterized by $A_1 = A_2 = A_3 = 0$. The transfer function of a fourth order Butterworth highpass is

$$B4(s) = \frac{s^4}{(s^2 + \sqrt{2-\sqrt{2}}s + 1)(s^2 + \sqrt{2+\sqrt{2}}s + 1)} \quad (7)$$

If we are interested in the group delay response of a Butterworth highpass we can look at the slightly more simple transfer function of a Butterworth lowpass which has the same group delay characteristics. Replacing s as in the previous section it can be expressed as

$$B4_{LP}(j\omega) = \frac{1}{1 + 2.613\,j\widehat{\omega} - 3.414\,\widehat{\omega}^2 - 2.613\,j\widehat{\omega}^3 + \widehat{\omega}^4}. \quad (8)$$

The general expression for the phase of the fourth order Butterworth lowpass filter is then

$$\beta = -\arctan\left(\frac{2.6131\,\widehat{\omega} - 2.6131\,\widehat{\omega}^3}{\widehat{\omega}^4 - 3.4142\,\widehat{\omega}^2 + 1}\right) \quad (9)$$

from which the group delay can be calculated as $\tau_g = -\mathrm{d}\beta/\mathrm{d}\omega$. The general expression for the group delay of a fourth order Butterworth filter (highpass or lowpass) can be found in eq. (20) in the appendix.

As can be seen in Fig.1, the maximum group delay occurs roughly at the cabinet resonance frequency. The group delay at resonance frequency is easily obtained by setting $\omega = \omega_0$ as

$$\tau_{max} \approx \tau(\omega = \omega_0) = \frac{3.695517}{\omega_0} = \frac{0.58816}{f_0}. \quad (10)$$

Hence, the approximate group delay maximum for a B4 alignment of interest can be determined quite simply. Another point of the group delay response can be estimated. The limit for $\omega \to 0$ is

$$\tau_g(0) = \frac{2.6131}{\omega_0} = \frac{0.4159}{f_0}. \quad (11)$$

With the knowledge of these two points of a group delay response, it is already possible to design a group delay equalizer. However, it has to be known that the box has a B4 alignment and the value f_0 is needed. Furthermore, the B4 alignment is only usable for drivers with $Q_{ts} \approx 0.4$, hence not all vented loudspeakers are designed using a B4 alignment.

2.3. Other alignments

Beside the B4 alignment there also exist further alignments like QB3 (quasi third order Butterworth), (S)C4 ((sub) fourth order Chebychev) and some more [18]. Their transfer functions include further design parameters that can be chosen to obtain desired response characteristics. Hence the determination of the group delay function for these type of filters is not possible in general.

Furthermore, a vented box is usually designed using one of these known alignments, but this is not strictly necessary, since the coefficients of the transfer function $G_v(s)$ can be set to any desired value as long as the design parameters as box size, tuning frequency and driver parameters allow.

Consequently, a more general approach is required to be able to equalize every loudspeaker.

3. DETERMINATION OF LOUDSPEAKER GROUP DELAY

If the alignment of the box is not known or the box is not designed corresponding to one of the known alignments, the parameters of the transfer function $G_v(s)$ can be determined by a measurement.

3.1. Acoustic measurement

The frequency response of any speaker can of course be measured by an acoustical measurement using an appropriate test system. However such a measurement requires a suitable (measurement-) microphone, a microphone preamp and software for signal generation and analysis. Such equipment is not always available, e.g. in a home environment.
Furthermore the results of an acoustical measurement depend heavily on the measurement room. Noise, reflections and standing waves can have a big influence on the results. Especially noise and reflections can cause peaks or ripple in the magnitude as well as in the phase response. Since the group delay is the derivative of the phase w.r.t. frequency, these unwanted disturbances influence the group delay measurement significantly.

3.2. Impedance measurement

The low-frequency behaviour of a loudspeaker can also be determined with an electrical impedance measurement [2]. In this case only a sine-generator, a voltmeter and an amperemeter are necessary. In many cases it should be possible to use standard multimeters, since even simple ones are dedicated to make measurements at 50 Hz and hence in the low audio frequency range. Furthermore, only the frequencies of three extreme values of the impedance response are needed, not the impedance values themselves. We have compared high-precision TRMS (HP 34401A) and simple non-TRMS multimeters without significant differences in this application.
From an impedance measurement the system compliance ratio α, the tuning factor h and Q_{ts} of the driver can be computed. With the knowledge of these values and an assumption for the box quality factor Q_l, the transfer function $G_v(s)$ and thus the low-frequency transfer characteristic of the loudspeaker is completely defined. In

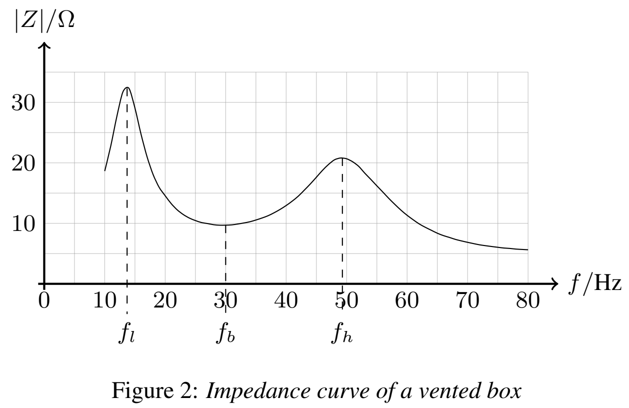

Figure 2: *Impedance curve of a vented box*

Fig.2 a typical impedance for a vented box can be seen. There are three frequencies of interest where f_b is the enclosure resonance frequency and a minimum of the impedance occurs. f_l is the frequency of the impedance maximum below f_b, and f_h is the frequency of the maximum above f_b. The resonant frequency of the speaker in the box f_{sb} can be computed as $f_{sb} = (f_h f_l)/f_b$. An alternative measurement method which requires blocking of the port is described in [19].
From the results of the impedance measurement, according to [4] the compliance ratio α can be computed as

$$\alpha = \frac{(f_h^2 - f_b^2)(f_b^2 - f_l^2)}{f_h^2 \, f_l^2}$$

Furthermore the tuning factor can be computed from the impedance measurement as

$$h = \frac{f_b}{f_s} \approx \frac{f_b}{f_{sb}}. \tag{12}$$

The free air resonance frequency f_s normally deviates only slightly from the resonance frequency of the built-in speaker f_{sb} [19], hence the influence of this approximation on the group delay will also be small.
With the knowledge of Q_{ts} all coefficients of the general transfer function can be determined. Bullock [5] gives an approximate formula for the total driver quality factor as

$$Q_{ts} = \left(\frac{1}{20\,\alpha}\right)^{\frac{1}{3.3}}. \tag{13}$$

For the box quality factor Q_l, a value of $Q_l = 7$ can be assumed. A slightly larger value could be applied for smaller boxes and a smaller value for larger ones.
With the knowledge of α, h, Q_l and Q_{ts} the coefficients of the transfer function a_1, a_2 and a_3 of the loudspeaker can be computed using equations (2) - (4). Hence the transfer function $G_v(s)$ and the resulting magnitude, phase and group delay responses can be determined independently of the used alignment. If the measurement is made directly at the terminals of a complete loudspeaker, the influence of the crossover network, which mainly influences Q_{ts} is already included in the results.
An advantage of this method is, that it directly yields a parametric description of the transfer characteristics, as only the three frequencies f_l, f_b and f_h have to be determined. Hence no smoothing is required as it would be the case with a direct acoustic measurement.

4. CORRECTION FILTER DESIGN

If the group delay is known, the next step is the design of the correction filter for compensation of the vented-box group delay at low frequencies. Because of the very small ratio of the lower cut-off frequency f_0 to the sampling frequency f_s, very long filters are needed in the case of FIR-filters to obtain a satisfactory frequency resolution. IIR-filters can work with a significantly lower amount of coefficients but will have high demands on the precision of the coefficients necessary for this task.
Consequently, if a suitable filter has been designed, the filtering process may also require a high resolution, i.e. a powerful processor in the case of a long FIR-filter or high precision in the case of an IIR-filter.
The computation of the filter coefficients can be challenging due to the above reasons. For example an optimization based method as described in [20] can be applied to design an allpass filter that approximates the phase response of the loudspeaker. However, this method may not directly give good results or stable filters in our

application. This is due to the fact that the phase is only approximated in a very narrow frequency band, for which the numerical conditions become an issue. Furthermore a suitable phase offset, which is not directly included in the optimization problem has to be chosen to ensure good approximation of the phase. We will show two alternative methods to design a correction filter.
The required equalization filter must have a negative group delay in a certain frequency range to "speed up" the signal or must introduce additional delay, to "slow down" signals in the complementary frequency range. In the second case an additional delay will be introduced into the signal path, which has to be considered, e.g. in live applications or audio/video synchronous tasks.

4.1. Equalization filters with negative group delay

The use of such filters is not directly possible, because filters showing negative group delay have a high-pass magnitude response [21]. The frequency range, in which the negative group delay occurs is then in the stopband of the filters. This would attenuate frequencies in the desired low frequency range and therefore would need an additional equalization (amplification) which would result in a poor signal-to-noise-ratio.
The use of allpass filters with negative group delay (which would be the filters with the desired characteristic in our application) is not possible. These filters are not stable because their poles would be located outside the unit circle.

4.2. Equalization filters with inverse delay

Such a filter should increase the group delay at all frequencies except the ones near resonance frequency of the cabinet, which could be achieved using allpass filters. This means that a large filter order has to be used to obtain low ripple in the group delay response for higher frequencies. Furthermore the fact, that the required delay can become quite high at typical audio sampling frequencies (some 1000 samples) the Q of the group delay for one allpass is very large since the poles resp. zeros have to be very close to the unit circle. This further increases the filter order needed to obtain a low ripple in the group delay response.
Two alternative methods to design such a filter are shown in the following.

4.2.1. FIR filter with unit magnitude response and inverse phase response

A frequency response function with a constant magnitude and arbitrary phase can be designed in the frequency domain directly. As a starting point, the transfer function of the loudspeaker $G_v(s)$ can be transformed to the discrete time domain via the bilinear transform to obtain $G_v(z)$. Then the impulse response $h_1(n)$ of this IIR filter can be computed for a desired number N of samples. The response can then be transformed to the frequency-domain using a discrete Fourier transform (DFT). In the next step, in the frequency domain the magnitudes can be set to an arbitrary value, e.g. unity if only a phase equalization is required. If the phase has to be equalized to exactly cancel the original phase and no magnitude equalization is desired, this is the only modification needed in the frequency domain. To ensure real coefficients of the filter in the time domain, it has to be ensured that the spectral values of a length N filter satisfy the relation

$$H_1(k) = H_1^*(N-k), \qquad k \neq 0. \tag{14}$$

After a transformation back to the time domain via an inverse DFT, we obtain the impulse response of the FIR filter having only the phase response of the speaker and unity gain for all frequencies. To obtain the final equalization filter with inverse group delay w.r.t the original, the impulse response has to be time reversed.
The disadvantage of this approach is the resulting computationally expensive long FIR-filter that finally does the group delay equalization. This can make a real time implementation e.g. on an embedded DSP-system difficult. A computation of the convolution in the frequency domain using overlap-add or overlap-save schemes would reduce the effort significantly, but requires quite long Fast Fourier Transforms (FFTs) which require more memory, increase the delay due to block processing and can decrease the precision on fixed-point systems.
An advantage of this method is, that more correction can be designed into this filter, e.g. magnitude equalization or highpass filtering for driver protection without increasing the computational effort of the filtering process. The data for magnitude equalization could be obtained via an acoustic measurement whereas the incorporation of predefined functions like subsonic filters would not require additional measurements.
A correction filter as described can be designed using MATLAB. Filters that are not directly based on the modelled frequency response of the speaker but can be tuned manually or selected using presets can be designed with a tool like rePhase [22].

4.2.2. Direct design of an allpass with the same group delay as the speaker and computation as a time-reversed IIR-filter

In this approach we first design an allpass-filter with a group delay approximating that of the loudspeaker. Mainly, we want to compensate for the delay introduced by the cabinet, which is a resonator and hence a second order system, whereas the whole speaker is modelled as a fourth order system. Hence, the approach is to assume that it is sufficient to design a resonator as a two-pole filter

$$H_R(z) = \frac{b_0}{(1 - re^{-j\omega_0}z^{-1})(1 + re^{j\omega_0}z^{-1})} \tag{15}$$

to mimic the group delay response of the box.
Two parameters, the radius r and the angles $\pm\omega_0$ of a pole pair $p_{1,2}$ have to be determined. This can be done based directly on parameters of the original (measured) group delay response. In this approach the resonance frequency of the box determines the angle of the poles of the correction filter. The radius of the poles determines the rate of change of the phase at the pole frequency and thus the maximum of the group delay.
The maximum magnitude response of a resonator which corresponds to the maximum group delay does not directly occur at the pole frequency ω_0 but also depends on the pole radius r and occurs at the frequency

$$\omega_r = \arccos\left(\frac{1 + r^2}{2r}\cos\omega_0\right) \tag{16}$$

[23]. This shift of the resonance frequency is due to the fact that we have poles at positive and negative frequencies to get real coefficients, i.e. a two-pole system for a single resonance. The pole at negative frequencies also has an influence on the response evaluated on the positive frequency axis in the upper half of the unit circle and vice versa. For values of r close to unity, the pole of the respective half-plane dominates and the dependency of the resonance frequency from the second pole can be neglected and thus

$\omega_r \approx \omega_0$. Hence we set the pole angles to

$$\omega_0 = \pm 2\pi \frac{f_b}{f_s}. \tag{17}$$

The required pole radius r can be derived from the phase response of the resonator. Here also both poles influence the desired group delay value, which makes the relation quite complicated. The expression for the group delay at ω_0 dependent on r is given in eq. (21) in the appendix. The expression in eq. (21) is quite unhandy and not easily to solve for r. A method for the determination of the required value of r is to compute it iteratively. A suitable starting point are the pole radii of the original transfer function $G_v(z)$.
In addition to the poles of the resonator, two zeros $z_{1,2}$ have to be added to obtain an allpass-system with a constant magnitude response. With the two zeros

$$z_{1,2} = \frac{1}{p^*_{1,2}} \tag{18}$$

we obtain the resulting transfer function of the allpass filter as

$$H_2(z) = \frac{(1 - \frac{1}{r}e^{j\omega_0}z^{-1})(1 + \frac{1}{r}e^{-j\omega_0}z^{-1})}{(1 - re^{-j\omega_0}z^{-1})(1 + re^{j\omega_0}z^{-1})}. \tag{19}$$

The zeros compensate the magnitude and add an additional delay of the same amount as that of the poles. Since $\omega_r = \omega_0$ is fixed, the poles have to be complex conjugates and the zeros directly result from the poles, r is the only parameter to be adjusted for the whole allpass equalization filter. Fig. 3 shows a pole-zero plot corresponding to the application example in the next section which shows the dimension of ω_0 and r.

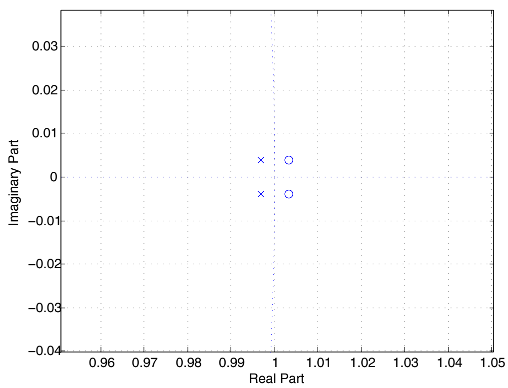

Figure 3: *Poles and zeros of the correction filter before time-reversal*

We now have a second-order recursive filter which approximates the group delay of the loudspeaker. To equalize the speaker, it has to be time reversed, which would normally lead to an unstable filter. This method for the design of the correction filter is not as exact as the approximation using a long FIR-filter on the basis of the measured group delay described in the section above. The advantage of this method lies in the significantly reduced computational effort required to run the filter in real-time.
The time-reversed low-order allpass $H_2(z)$ can be realized efficiently as an IIR-filter using the structure proposed in [24]. The

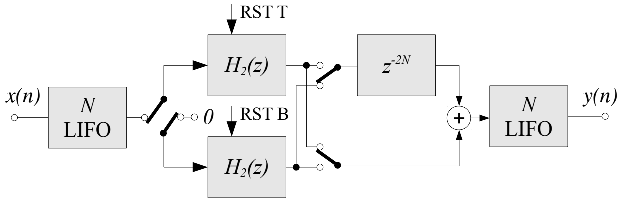

Figure 4: *Block diagram of time-reversed filter implementation from [24]*

block diagram of this filter is shown in Fig.4. Using this method, data is buffered for a number of N samples using a Last In First Out (LIFO) buffer. The output of the buffer is a time-reversed version of the input signal and sent through the second order allpass filter, which requires only 5 multiplies and 6 additions per output sample. Due to the long time constants it may be necessary to implement the filter in double precision which would increase the computation time roughly by a factor of four, which is still much less than a FIR implementation requires. The result is then again time-reversed by a second length-N LIFO-buffer and given to the output. To account for adjacent blocks an overlap-add scheme is applied. Computing the equalization filter as a time-reversed IIR-filter significantly reduces the required computational effort compared to an FIR implementation. However, there is no free lunch and the drawback is, that the delay is increased to $2N$ samples and the memory requirements to $4N$ samples. This would allow to run the equalization filter on quite simple platforms, provided, that they have enough memory. The additional delay may not pose a problem if just a music playback situation is considered.
Another advantage of this approach is, that the delay of the equalization filter can be changed quite easily just by changing pole radius r and re-computing the 5 filter coefficients of $H_2(z)$. The computation of an inverse DFT and time inversion is avoided. This would allow to implement an adjustable delay on an embedded system.
Both approaches, the FIR-filter and the time-reversed IIR-filter use a truncated impulse response of a recursive system as a correction filter. The required length N of this impulse response is of course dependent on the sampling frequency f_s and should be chosen to provide a minimum frequency resolution $\Delta f = f_s/N$ of about 5 Hz. This results in a value of $N \geq 8820$ for $f_s = 44.1$ kHz.
When using an FIR-filter this would mean 8820 multiply and accumulate (MAC) operations per output sample in contrast to about 50 operations and some overhead for the buffering operations when using the time-reversed IIR approach in double precision.

5. APPLICATION EXAMPLE

The following examples show the application of the proposed correction technique to a commercial HiFi loudspeaker (JBL TI5000). This speaker shows an electrical impedance at the loudspeaker terminals as shown in Fig.2. The three frequencies of interest for this speaker are $f_l = 13.8$ Hz, $f_b = 30$ Hz and $f_h = 49$ Hz. The computed total driver quality factor is $Q_{ts} = 0.31$ and the resonance frequency $f_{sb} = 22.6$ Hz. These values are in good accordance with the data given by the manufacturer with $f_b = 30$ Hz, $f_s = 24$ Hz and $Q_{ts} = 0.29$. From the measured frequency values the additional parameters are computed as $\alpha = 2.32$ and $h = 1.33$, which are reasonable values for a QB3 alignment.

With these values the coefficients of the transfer function $G_v(s)$ can be computed. The original magnitude and group delay responses of the speaker as computed from the data of the impedance measurement are shown in Fig.5. The maximum value of the group

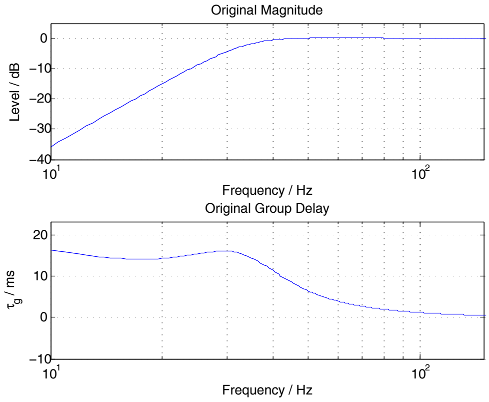

Figure 5: *Original Magnitude and group delay response of the loudspeaker*

delay τ_{gmax} is about 16.5 ms at a frequency of 29.5 Hz, which is close to the measured cabinet resonance frequency of 30 Hz. The -3 dB corner frequency of the system is at about 32.5 Hz.
The resulting pole radii of the discrete-time fourth-order highpass $G_v(z)$ are: 0.99803, 0.99803, 0.99853 and 0.99454. A correction filter has been designed as described in 4.2.2. The pole angles are chosen as $\omega_0 = 2\pi(f_b/f_s)$ with $f_b = 30$ Hz and $f_s = 44.1$ kHz and the pole radii r were determined iteratively as $r = 0.9968$. This leads to the pole-zero configuration as shown in Fig. 3.
In Fig.6 the frequency response of the time-reversed correction filter is shown. The group delay correction is not exact, as expected because the box is a fourth order system and the correction filter a second order system and only models the cabinet resonance. Furthermore the magnitude is unity because of the additional zeros placed at the mirror position of the poles. For evaluation of

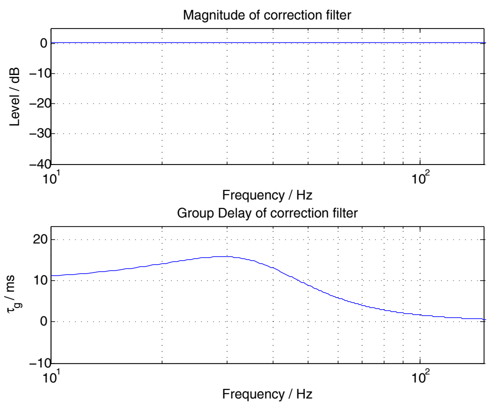

Figure 6: *Magnitude and group delay response of the correction filter before time-reversal*

the equalization performance, the group delay of the correction filter has been subtracted from the original group delay of the loudspeaker. The result is shown in Fig.7. The correction filter does not affect the magnitude response of the speaker but reduces the group delay error of the box in the audible frequency range significantly. The group delay error is about 4 ms at a frequency of 10 Hz, where the magnitude is already at about -35 dB w.r.t. the passband and about -2.4 ms at a frequency of 47 Hz. This error of -2.4 ms is much smaller than the original group delay of 9.3 ms at this frequency before equalization. In [15] it is stated that a delay below 1-2 ms will practically never be noticed and 3-5 ms errors are safe for most program material. At the cabinet resonance frequency of 30 Hz, the group delay is zero as expected.

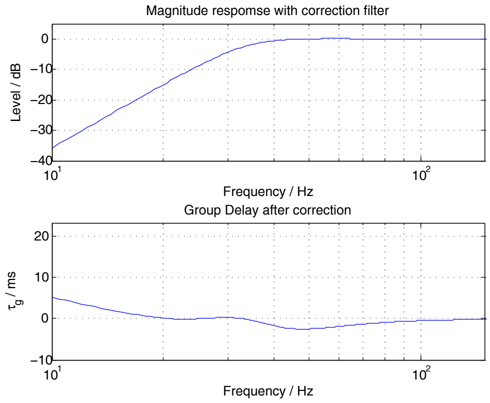

Figure 7: *Magnitude and group delay response of the corrected speaker*

The performance of the filter can be fine tuned by manually adjusting ω_0 and r to further reduce the errors or to adjust the equalization to personal preferences.

5.1. Results

The result has been evaluated in an informal listening test, where the correction was clearly audible for all participants. The low-frequency reproduction gets tighter and more defined. Rhythmic instruments like bassdrums have a better coherence of bass and subbass frequencies and thus are fusing more into one sound. Due to the change introduced by the equalization, the resulting sound is also a little unusual since the listener is in most cases used to listening to uncorrected speakers for a long time.
Another observation is, that the crestfactor of the output signal of the correction filter can change due to the phase shifts in the low-frequency range. To avoid clipping, the level of the output signal may have to be reduced or limited according to the capabilities of the signal processing system.

6. CONCLUSIONS

A way to determine the transfer function and thus the group delay of a vented box in a simple applicable way via an electrical impedance measurement has been shown. The group delay deficiencies of the speaker can be equalized with an FIR-filter, into which further equalization can be incorporated. This method can give very accurate equalization but is computational demanding.
The correction filter can also be designed as a time-reversed allpass by choosing the appropriate resonance frequency and pole radii of

a second order resonator whose magnitude is then corrected with additional zeros. This approach does not account for all sources of unwanted group delay and but delivers good results. Furthermore it allows for a parametric filter design and thus an implementation of a simple real-time control of the delay. Additionally, it reduces the computational load of the filtering process significantly with the cost of introducing some additional delay into the signal path.

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8. APPENDIX

The general expression for the group delay of a fourth order Butterworth filter (highpass or lowpass) is

$$\tau_{gB4}(\omega) = \frac{9.28986\,\omega_0\,\omega^6 + 3.84786\,{\omega_0}^3\,\omega^4 + 3.84786\,{\omega_0}^5\,\omega^2 + 9.28986\,{\omega_0}^7}{3.55511\,\omega^8 - 0.000386\,{\omega_0}^2\,\omega^6 + 0.000635\,{\omega_0}^4\,\omega^4 - 0.000386\,{\omega_0}^6\,\omega^2 + 3.55511\,{\omega_0}^8}. \tag{20}$$

The expression for the group delay introduced by a pole pair at $\pm\omega_0$ with both poles having the radius r is

$$\tau_g(r) = -\frac{3r^3\sin^2(2\omega_0) + \left(6r^3\cos^2(\omega_0) - 2r^2\right)\cos(2\omega_0) + \left(2r - 4r^2\right)\cos^2(\omega_0) - 2r^4}{2r^3\sin^2(2\omega_0) + (4r^3\cos^2(\omega_0) - 2r^2)\,\cos(2\omega_0) + (4r - 4\,r^2)\,\cos^2(\omega_0) - r^4 - 1}. \tag{21}$$