DISTORTION AND PITCH PROCESSING USING A MODAL REVERBERATOR ARCHITECTURE

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ABSTRACT

A reverberator based on a room response modal analysis is adapted to produce distortion, pitch and time manipulation effects, as well as gated and iterated reverberation. The so-called "modal reverberator" is a parallel collection of resonant filters, with resonance frequencies and dampings tuned to the modal frequencies and decay times of the space or object being simulated. Here, the resonant filters are implemented as cascades of heterodyning, smoothing, and modulation steps, forming a type of analysis/synthesis architecture. By applying memoryless nonlinearities to the modulating sinusoids, distortion effects are produced, including distortion without intermodulation products. By using different frequencies for the heterodyning and associated modulation operations, pitch manipulation effects are generated, including pitch shifting and spectral "inversion." By resampling the smoothing filter output, the signal time axis is stretched without introducing pitch changes. As these effects are integrated into a reverberator architecture, reverberation controls such as decay time can be used produce novel effects having some of the sonic characteristics of reverberation.

1. INTRODUCTION

The "modal reverberator" algorithm [1, 2] proposed a unification of perspectives on room reverberation analysis with the goals of synthetic reverberation. While room reverberation has long been analyzed from the viewpoint of modal analysis [3, pg. 172, ff.], [4, pg. 576, ff.], artificial reverberation is typically synthesized using structures such as delay networks or convolution which attempt to reproduce time domain features of the room response [5]. By contrast, the modal reverberator implements room modes directly as the sum of parallel resonant filters.

The parallel architecture of the modal reverberator provides explicit, interactive control over the parameters of each mode, allowing accurate modeling of acoustic spaces, as well as movement within them and morphing among them. Here, we extend this structure to allow manipulation of the mode responses in ways that lead to alternative implementations of audio effects such as distortion and pitch shifting, and to novel effects which integrate reverberation into nonlinear processes.

Schroeder and Logan's seminal 1961 article " 'Colorless' Artificial Reverberation" [6] introduced the comb and allpass filters as powerful building blocks for designing artificial reverberators and launched a flurry of research and commercial activity in the field. Their perspective led directly to artificial reverberation algorithms using digital waveguide and physical modeling approaches [7, 8], multi-feedback unitary systems [9], and the Feedback Delay Network (FDN) framework [9–12]. This family of methods, based on networks of delay lines and filters, is still considered state of the art today [5].

Although Schroeder and Logan pioneered the use of delay lines in mimicking a room's response, Schroeder was an experienced researcher of the modal properties of rooms [13, 14] and they situated their work in the frequency domain. Their article [6] opens: "A room can be characterized by its normal modes of vibration." This perspective is standard in room acoustics [4, 15] and musical instrument modeling [16], and in fact modal thinking about room acoustics has its roots in antiquity. Blesser traces a history of interactions between space, music, and culture [17], including a history of *acoustical vases*, Helmholtz resonators set up in amphitheaters to change their reverberant properties—the tradition was well-established when Vitruvias reported on it in 30 B.C. [18] and continued into medieval times [19].

In artificial reverberation research, ironically, the modal perspective has never quite risen above a theoretical framing device; mode responses are not implemented directly. Blesser gives a thorough review of eigentone statistics [17] before concluding: "The eigentone model is much less useful than the random echo model for many reasons."

In fact, in FDNs and related frameworks, audible modes can be considered undesirable artifacts. Moorer reports on "annoying ringing" in Schroeder-style cascaded allpass reverberators [20]. Griesenger [21] describes Stautner's approach (which is widely used) to controlling "unpleasant resonant behavior": time-varying delays and mild feedback [10]. Jot writes of "unnatural resonances" which sound "metallic" [11, 22]. In the discourse of FDN limitations, late-field resonances are considered to be artifacts much like limit cycles [20, 23].

In the development of artificial reverberation algorithms, many techniques created out of practical necessity are repurposed and exaggerated to artistic effect. For instance, consider that early work on feedback systems used frequency shifting to avoid howling in public address systems [24, 25]. Artificial reverberation algorithms such as Sean Costello's ValhallaShimmer [26] and Vesa Norilo's "Vectored Time Variant Comb Filter" [27] have repurposed this practical tool as an artistic effect. Consider also the concept of modulating delay lines in FDNs to break up patterns of regular echos, a technique well represented in the literature [10, 12, 21] and found in commercial products such as the Lexicon 224 [28, 29]. Now, algorithms may use more extreme versions of delay-line modulation to achieve versions of, e.g., chorus and flangers within feedback loops [27]. Embedded "Slinky" paths in FDNs can be considered an exaggeration of natural allpass characteristics along a propagation path [30].

In this work, we propose extensions to the modal reverberator algorithm [1, 2] that parallel this development, emphasizing the algorithm's potential as a platform for new musical effects. Like extended FDNs, a "modal effects processor" framework yields classes of novel musical effects. In §2 the modal reverberator is reviewed. In §3, classes of modal effects are presented, including gating and envelope processing (§3.1), time stretching (§3.2), pitch manipulation (§3.3), and distortion processing (§3.4).

2. MODAL REVERBERATOR

Acoustic spaces and vibrating objects have long been analyzed in terms of their normal modes [4, 16]. The impulse response $h(t)$ between a pair of points in the system may be expressed as the linear combination of mode responses,

$$h(t) = \sum_{m=1}^{M} h_m(t), \tag{1}$$

where the system has M modes, with the mth mode response denoted by $h_m(t)$, t being the discrete time sample index. The system output $y(t)$ in response to an input $x(t)$, the convolution

$$y(t) = h(t) * x(t), \tag{2}$$

is then seen to be the sum of mode outputs

$$y(t) = \sum_{m=1}^{M} y_m(t), \quad y_m(t) = h_m(t) * x(t), \tag{3}$$

where the mth mode output $y_m(t)$ is the mth mode response convolved with the input. The modal reverberator simply implements this parallel combination of mode responses (3), as shown in Fig. 1.

In general, mode responses $h_m(t)$ are complex exponentials, each characterized by a mode frequency ω_m, mode damping α_m and complex mode amplitude γ_m,

$$h_m(t) = \gamma_m \exp\{(j\omega_m - \alpha_m)t\}. \tag{4}$$

The choice of complex, rather than real, mode responses is made here for clarity of presentation and to suggest the implementation structures described below. A real response would be formed by a combinations of conjugate responses; here a proportional result is formed by taking the real part of each complex mode response. A stereo effect can by obtained by taking the imaginary part as a second channel.

The mode frequencies and dampings are properties of the room or object. They describe, respectively, the mode oscillation frequencies and decay times. The mode amplitudes are determined by the sound source and listener positions (driver and pick-up positions for an electro-mechanical device), according to the mode spatial patterns.

Note that even for short reverberation times of just a few hundred milliseconds, the mode responses $h_m(t)$ are very resonant, and last many thousands of samples at typical audio sampling rates. In implementing the mode filters, therefore, numerically stable methods must be used. One such method is the phasor filter [31, 32], in which each mode filter is implemented as a complex first-order update,

$$y_m(t) = \gamma_m x(t) + e^{(j\omega_m - \alpha_m)} y_m(t-1). \tag{5}$$

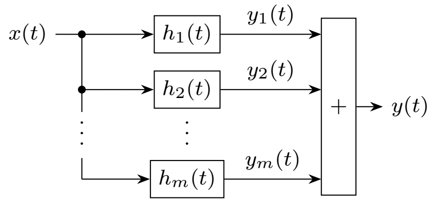


Figure 1: *Modal Reverberator Architecture. The modal reverberator is the parallel combination of resonant filters matched to the modes of the system.*

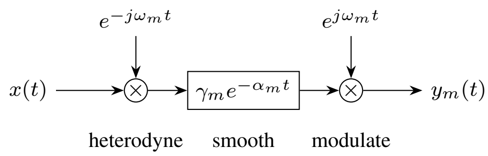


Figure 2: *Mode Response Implementation. The mode response may be implemented as a cascade of heterodyning, smoothing and modulation operations.*

Another approach is to rearrange the mode response convolution,

$$\begin{aligned} y_m(t) &= \sum_{\tau} \gamma_m e^{(j\omega_m - \alpha_m)(t-\tau)} x(\tau) & (6)\\ &= e^{j\omega_m t} \sum_{\tau} \gamma_m e^{-\alpha_m (t-\tau)} \left[e^{-j\omega_m \tau} x(\tau) \right]. & (7) \end{aligned}$$

Here, the mode filtering is implemented by heterodyning the input signal to dc to form a baseband response, smoothing this baseband response by convolution with an exponential, and modulating the result back to the original mode frequency,

$$y_m(t) = e^{j\omega_m t} \cdot \left(\gamma_m e^{-\alpha_m t} * \left[e^{-j\omega_m t} x(t) \right] \right). \tag{8}$$

This process is shown in Fig. 2. The heterodyning and modulation steps implement the mode frequency, and the smoothing filter generates the mode envelope, in this case an exponential decay.

Using this architecture, rooms and objects may be simulated by tuning the filter resonant frequencies and dampings to the corresponding room or object mode frequencies and decay times. The parallel structure allows the mode parameters to be separately adjusted, while the updates (5) or (8) provide interactive parameter control with no computational latency.

Three design approaches are suggested in [2] for setting the mode parameters: behavioral, analytical, and perceptual. The behavioral approach fits mode parameters to system measurements. The analytic approach derives mode parameters from the physics of the system. The perceptual approach selects mode parameters according to desired wet response equalization and, say, early and late decay times. An example design is shown in Fig. 3, in which a modal reverberator architecture is fit to the measured impulse response of a classroom. For this design 1605 modes were used, and the modal system response was a close perceptual match to that of the measurement.

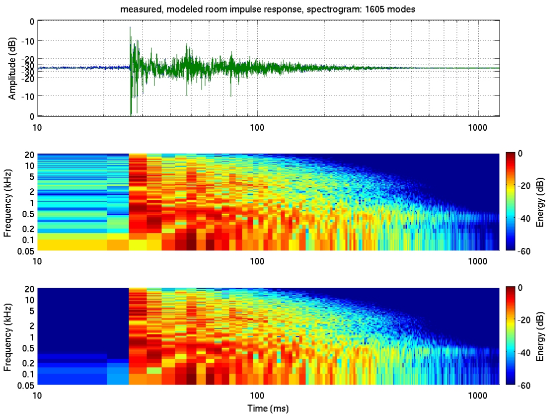
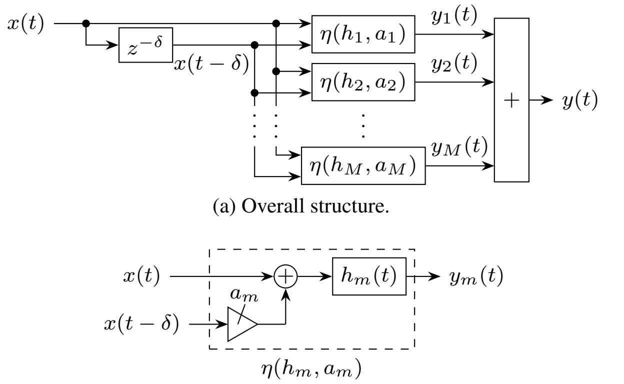


Figure 3: *Example Modal Reverberator Design. Measured and modeled room impulse responses for a roughly 10 m×12 m×5 m classroom, Room K217 at The Knoll, Stanford University, are overlaid (top), and the corresponding spectrograms shown (middle, modeled; bottom, measured).*



(a) Overall structure.

(b) Detail of the mth band.

Figure 4: *Modal Gated Reverberator Signal Flow Architecture, including (a) overall structure and (b) detail of an individual band.*

3. MODAL EFFECTS PROCESSOR

The modal reverberator lends itself to effects processing through its parallel architecture and dense, narrowband mode responses. In the following, we present four categories of effects, all provided by manipulating different blocks of the mode response processor shown in Fig. 2:

- gating and envelope processing,
- time stretching,
- pitch manipulation, and
- distortion processing.

Reverberation envelope effects such as gating and iterated convolution may be achieved by manipulating the smoothing filter response. Time stretching without pitch shifting is possible by resampling the smoothing filter output. Pitch manipulation effects such as pitch shifting and spectral "inversion" are available by using different sinusoid frequencies for the heterodyning and modulation steps. Finally, distortion effects may be generated by distorting or substituting for the modulation sinusoid waveform.

Since these effects are integrated into a reverberation architecture, their sonics are different than their standard counterparts when the mode decay times are longer than a few hundred milliseconds. The result is a unique effect with sonic qualities of both the standard effect and reverberation.

3.1. Reverberation Envelope Effects

We first describe reverberation envelope effects.

Gated reverberation is a reverberation effect in which the reverberation response onset is rapidly forced to zero after a short period of time, say 250 ms. The effect was popularized by Phil Collins in the 1980s (e.g. [33]). One approach, used by the AMS RMX-16, one of the first digital reverberators and popular in the 1980's [34], implements a system impulse response which decays very rapidly after a given point in time.

In the modal architecture discussed here, the individual mode responses are decaying exponentials, and may be "switched off" after a specified delay using a truncated IIR (TIIR) technique [35]. A first-order mode filter impulse response may be truncated after a given delay by subtracting from its input an appropriately scaled, delayed version of the input. Stated mathematically, the mth mode filter impulse response

$$h_m(t) = e^{(j\omega_m - \alpha_m)t} \tag{9}$$

can be made zero starting at a delay δ by replacing the input $x(t)$ with

$$x(t) - e^{(j\omega_m - \alpha_m)\delta} x(t-\delta). \tag{10}$$

This is implemented in the signal flow architecture shown in Fig. 4. Note that the input signal delay is implemented just once, outside the modal reverberator structure, and the scaling, $\exp\{(j\omega_m - \alpha_m)\delta\}$, which varies from mode to mode, is implemented locally (Fig. 4b).

As an example, consider the dry and processed guitar track snippets shown in Fig. 5. Here, the reverberator employed 2048 modes, with decay times ranging from 2500 ms in the mid frequencies to 200 ms in the high frequencies. The gate time δ was set to 290 ms. Transients in the guitar track reverberated for only a short time before truncation.

We suggest two variations: An interesting "gated cathedral" sound results when a long reverberation time is used, and the response isn't fully truncated. This is accomplished by slightly reducing the magnitude of the scale factor, e.g., to $0.95\exp\{(j\omega_m - \alpha_m)\delta\}$. Another artistic effect forms groups of modes, with different groups having different gate times. An example impulse response of such a system is shown in Fig. 6.

This TIIR approach may be used with higher order mode response filters. For instance, repeated pole filters having N poles and impulse response onsets roughly proportional to t^{N-1} can be truncated to generate a "reverse reverberation" effect. To implement such filters, the structure of Fig. 4 can be augmented with additional delay lines having lengths equal to integer multiples of δ, up to $N\delta$. Finally, reverse reverberation can also be implemented using the TIIR approach with a growing exponential mode envelope.

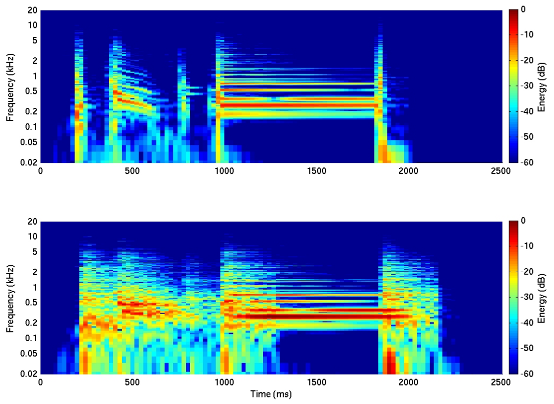


Figure 5: *Modal Gated Reverberator Processed Guitar Track. Dry (top) and processed (bottom) guitar track snippets. Note the truncation of reverberated transients.*

Another reverberation envelope effect is iterated reverberation, the repeated application of a reverberant impulse response, inspired by Alvin Lucier's piece "I am sitting in a room" [36]. Since the mode responses are orthogonal, the order-N iterated convolution of the system response $h(t)$,

$$h^{*N}(t) = \underbrace{h(t) * h(t) * \cdots * h(t)}_{N \text{ responses}}, \tag{11}$$

is the sum of the mode response iterated convolutions,

$$h^{*N}(t) = \sum_{m=1}^{M} h_m^{*N}(t). \tag{12}$$

The mode response iterated convolutions may be implemented using the approach of (8), in which the heterodyning and modulation operations are left unchanged and the mode envelope filter is cascaded with itself (i.e., iterated) N times. Doing so produces a mode envelope proportional to $t^{N-1}\exp\{-\alpha_m t\}$, which provides a delayed onset of late field energy, peaking at a time $(N-1)/\alpha_m$.

Additional reverberation envelopes include delayed onset and two-stage decays, as would be appropriate for modeling coupled spaces such as a box in an opera hall, and as described in [37, 38]. These can be implemented in the context of the modal reverberator structure by design of the mode envelope filter in a manner similar to that described in [38]. Alternatively, a two-stage decay may be implemented by having some modes take on a large amplitude and decay quickly while other modes have a smaller amplitude and decay slowly.

3.2. Time Stretching Effects

In the modal reverberator structure of Fig. 1 and Fig. 2, a reverberated signal is constructed from its mode responses, each of which is generated by applying its mode envelope to a sinusoid at its mode frequency. If the mode decay times are short (less than, say, a couple hundred milliseconds) and the mode energies are inversely proportional to the frequency density of nearby modes, then the processed audio will sound much like the input. In this case, if all of the mode envelopes are resampled to a stretched time axis, then the resulting signal will be a time-stretched version of the input.

As an example, Fig. 7 shows spectrograms of the dry guitar track of Fig. 4 time stretched in this way by factors of 0.25, 1, and 4. Note that the spectra are similar under the appropriate time axis scalings. While the timbers of the time-stretched signals match well, there are differences between the dry input and the slightly reverberated signal generated without resampling the mode envelope. Mainly, transients are less crisp.

One other artifact appears, a subtle beating or tremolo heard during sustained notes. Using a second-order smoothing filter, say the first-order filter repeated, effectively eliminates the problem. The examples of Fig. 7 were generated with such a second-order mode envelope. Finally, as the smoothing filter output has relatively low bandwidth, linear interpolation is expected to be sufficient for resampling, and was used to generate the time stretched signals of Fig. 7.

The time stretching can vary with time so as to compress or expand the time axis of different sections of the signal by different amounts. In addition, the parallel structure makes it simple to vary the time axis modification over frequency, for instance, having low frequencies time expanded and high frequencies time compressed.

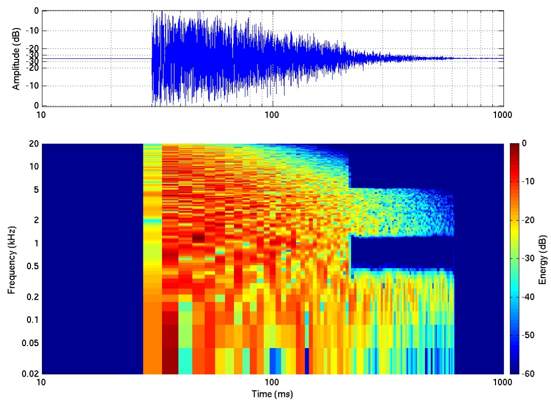


Figure 6: *Modal Gated Reverberator Impulse Response. The impulse response (top) and corresponding spectrogram (bottom) are shown for a modal gated reverberator with a gate time of 180 ms for certain frequency bands, and 580 ms for others. Logarithmic time axes are used.*

3.3. Pitch Manipulation Effects

Recall that the mode response can be thought of as the cascade of heterodyning, smoothing, and modulation operations, as shown in Fig. 1. Now consider replacing each modulation frequency ω_m with one shifted by σ half steps,

$$\nu_m = 2^{\sigma/12}\omega_m, \tag{13}$$

as seen in Fig. 8. Doing so will shift the pitch of the output relative to the input. This is illustrated in Fig. 9, in which a guitar track is shifted by -2, 0, and 2 octaves. Note that the shifted spectrograms

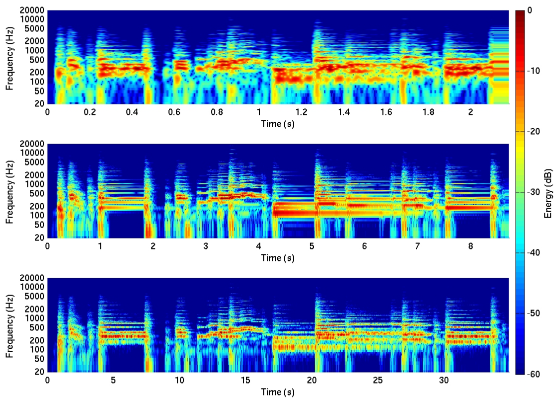


Figure 7: *Time Stretched Guitar Track. Spectrograms for a guitar track, processed using a 2400-mode processor with randomly selected, exponentially distributed mode frequencies in the range 20 Hz to 20 kHz and having 200 ms decay times, is shown with the mode envelope filter output sampled at rates of* 4 *(top),* 1 *(middle), and* $1/4$ *(bottom) of the system sampling rate of 48 kHz. Note the different time axes.*

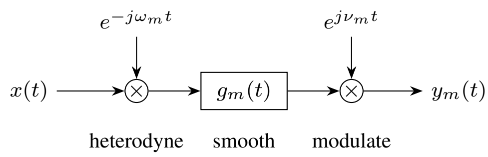


Figure 8: *Pitch Shifting Mode Response Implementation.*

are well matched, save the constant frequency resolution of the spectrogram.

As in the case of the time stretching effect above, we suggest using higher order mode envelope filters to eliminate tremolo-type artifacts. The signals shown in Fig. 9 were prepared with three iterations of the first-order response (i.e., four-pole filters) (4), though one iteration works well. There is a trade-off involving the mode decay time: With a short decay time, the mode envelope filter has a wide bandwidth, resulting in beating between adjacent modes. With a long decay time, the beating is eliminated, but the output will take on a reverberant quality, which might be unwanted.

In the example of Fig. 9, we used 200 ms decay times, and 2400 exponentially distributed mode frequencies. We heard little if any difference between deterministically and randomly generated mode frequencies. We used random mode phases $\angle\gamma_m$, uniformly distributed on the interval $[0, 2\pi)$, so that the system response to a pulse would lack structure, and therefore reduce temporal artifacts. Finally, note that modes shifted up or down in frequency outside the audio band need not be computed.

Since the modulation is computed on a sample-by-sample basis, the pitch shift may be changed on a sample-by-sample basis. Since the modes are independent, different pitch shifts may be applied to different modes. Fig. 10 shows a pitch shift which drifts upward over time, and for input frequencies above about 200 Hz, develops a vibrato having an increasing rate. Also shown in Fig. 10

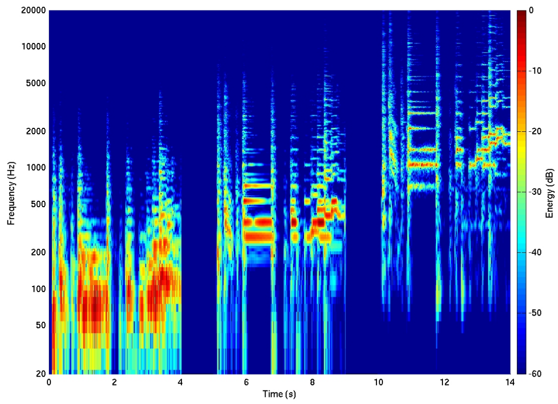


Figure 9: *Pitch Shift Example. Spectrograms are shown for a guitar track processed with a modal reverberator having 2400 modes and a 200 ms decay time, and pitch shifted by factors of* 4 *(top),* 1 *(middle), and* $1/4$ *(bottom).*

is a signal processed using the same pitch trajectory, but with a 1.5 s-long reverberation in the mid frequencies. In this case the mode dampings were kept constant, and the reverberation time as a function of frequency $T_{60}(\omega)$ moves with the pitch shift. To keep the reverberation time independent of the pitch shift, the mode dampings could be set according to the instantaneous pitch shifted frequencies, $T_{60}(2^{\sigma_m(t)/12}\omega_m)$.

Other pitch effects may be produced by shifting different mode frequencies by different amounts. A number of strategies for generating the modulation frequencies have been tried, including permuting the mode frequencies, generating random frequencies, and controlling the distance from the heterodyne frequencies to a quantized set of frequencies.

Another choice is a spectral "inversion," formed by inverting the mode frequencies about a center frequency, ω_c, and applying a frequency shift,

$$\nu_m = 2^{\sigma/12}\frac{\omega_c^2}{\omega_m}. \tag{14}$$

The effect creates different harmonic relationships among the partials present, as seen in the example of Fig. 11.

3.4. Distortion Effects

The modal reverberator architecture can be integrated with a distortion process by distorting each mode response before mixing to form the output, as shown in Fig. 12. Alternatively, groups of modes may be mixed and distorted together, as shown in Fig. 13.

With the modes individually distorted, or with the mode frequencies in a given group having harmonic relationships, the distortion produced will be free of dissonances from intermodulation products. This distortion has a different sonic character than is typical, producing a distorted sound while maintaining the harmonic structure of the dry track. There are a number of pop songs, including "Something About You" by Boston [39] and "God Save the Queen" by Queen [40], which use this type of effect with guitar, and presumably achieved by multitracking, building up chord sequences from single-note lines of music. (Such an effect could

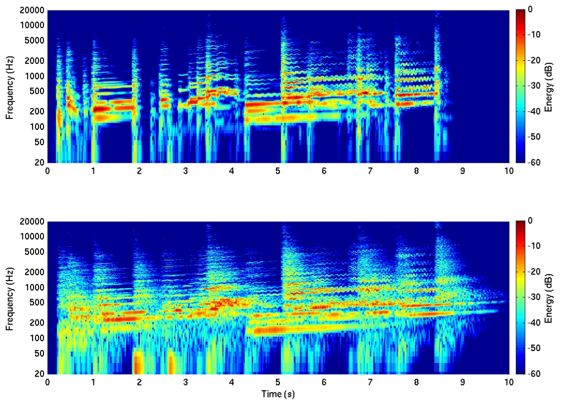


Figure 10: *Pitch Drift Example. The spectrogram of the guitar track of Fig. 9 processed according to a drifting, modulating pitch shift is shown (top), along with a version processed with the same pitch shift trajectory, but with a reverberation time of about 1.5 s (bottom). Note that since the mode dampings are left constant, the reverberation time drifts in frequency with the pitch shift.*

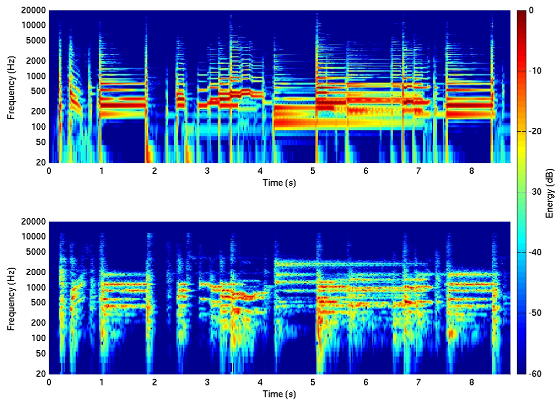


Figure 11: *Spectral Inversion Example. Spectrograms are shown for a dry guitar track (top) and a processed version (bottom), in which mode heterodyning and modulation frequencies are inverses of each other about 440 Hz.*

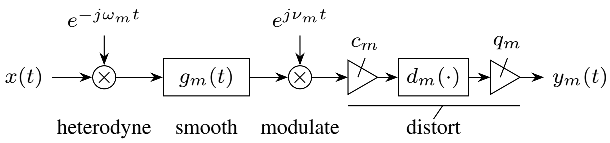


Figure 12: *Distortion Effect on One Mode.*

also be achieved using a hexaphonic pick-up, and having a separate distortion process on each string's output.) Modal distortion can be considered a further exaggeration of this method—rather than distorting individual notes, each mode response produced by a signal may be distorted independently.

Though it may be computationally costly to distort modes individually or in narrow-bandwidth groups, the need for upsampling to avoid aliasing may be eliminated, thus reducing the cost. To do so, consider a distortion function $d_m(w)$ expressed as a power series in w. On a per-mode or per-mode-group basis, $d_m(w)$ can be designed to produce output without aliasing by implementing only those power series terms which would produce harmonic distortion below the Nyquist limit. For instance, for modes having frequencies above half the Nyquist limit, only the constant and linear terms would be included; for modes having frequencies between half and one third the Nyquist limit, only the constant, linear, and quadratic terms would be included; and so on. (For a discussion of distortion and aliasing, see [41].)

An example distortion process output is shown in Fig. 14, with the guitar track used previously distorted in the presence of both short and longer reverberation times.

Another approach is to substitute periodic waveforms, say sawtooth or wavetable-based waveforms, for the sinusoidal modulators $\exp\{j\nu_m t\}$. Unlike the distortion from memoryless nonlinearities, this approach provides distortion that is independent of signal amplitude.

4. SUMMARY

In this work, we described extensions to the modal reverberator algorithm that produce audio effects in four categories: reverberation envelope control, time stretching, pitch manipulation, and distortion processing. This was achieved by implementing the resonant mode filters as cascades of heterodyning, smoothing, and modulation steps, and manipulating aspects of the smoothing and modulation operations.

The modal processor signal flow architecture is in some ways similar to that of a phase vocoder [42, 43], having a parallel bank of analysis, processing and synthesis operations. A key difference, however, is that the modal processor's filterbank center frequencies and bandwidths (i.e., dampings) are often chosen according to a desired acoustic space, rather than as the filters associated with a given Fourier transform length and window. This allows modal processor effects such as pitch shifting and distortion to incorporate reverberation, while providing relatively artifact-free effects when the mode decay times are short.

The parallel nature of the modal architecture allows different effects or no effect to be applied on a per mode or per mode group basis. The sample-by-sample processing allows continuous control of all effects parameters without latency and with no blocking artifacts.

Acknowledgement

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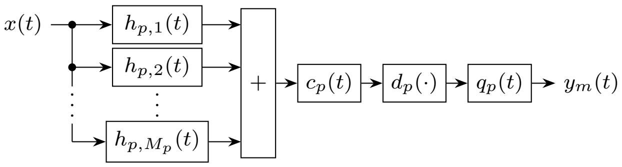

Figure 13: *Distortion Effect on a Group of Modes.*

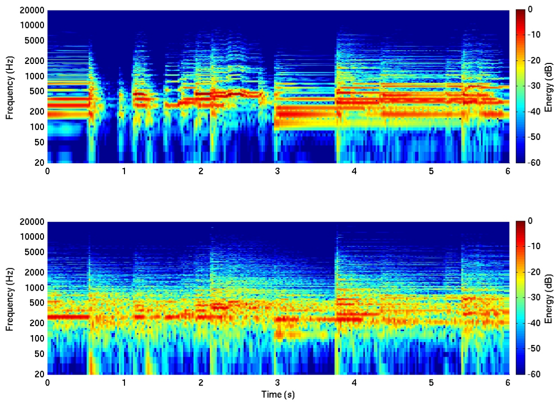

Figure 14: *Distortion Example. Spectrograms are shown for the guitar track used previously, processed with a modal distortion having decay times of 200 ms (top) and about 2 s (bottom).*

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