GROUP DELAY-BASED ALLPASS FILTERS FOR ABSTRACT SOUND SYNTHESIS AND AUDIO EFFECTS PROCESSING

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ABSTRACT

An algorithm for artistic spectral audio processing and synthesis using allpass filters is presented. These filters express group delay trajectories, allowing fine control of their frequency-dependent arrival times. We present methods for designing the group delay trajectories to yield a novel class of filters for sound synthesis and audio effects processing. A number of categories of group delay trajectory design are discussed, including stair-stepped, modulated, and probabilistic. Synthesis and processing examples are provided.

1. INTRODUCTION

Allpass filters are of particular interest for audio processing because they are defined to have a unit magnitude frequency response and exhibit time delays that vary with frequency. As passive, dispersive filters, they are found in a wide range of audio applications. For example, allpass filters are found in physical modeling of stiff strings and percussion [1,2], guitar bodies [3], pianos [4,5], and bells [6]. They are used for interpolation [7, 8] and decorrelation [9, 10]. The dispersive qualities of allpass filters are also useful for artificial reverberation [11] and spring modeling [12].

Allpass filters can be used for system measurement and identification [13, 14]. They can be found in shelving and equalization filters [15,16] as well as warped filters such as [17]. They have also been used in loudspeaker crossovers for time-alignment [18,19].

In a more abstract sense, one can find allpass filters in a range of audio effects. Flanging, phasing, and chorus effects [20–22] can be implemented with allpass filters, and [23–25] have used allpass filters for distortion processing. Recently, [26] has used allpass filters for peak limiting. Further developing the work of [27], [28, 29] have shown uses of allpass filters for abstract sound synthesis by means of spectral delays and [30, 31] has shown methods for using allpass filters for distortion effects.

In this paper, we present a method for designing allpass filters from their group delay. We show that for an allpass filter, the group delay can be interpreted as a trajectory of frequency-dependent time delays, used here primarily for artistic effects. This group delay can be arbitrarily formed so long as each frequency has exactly one associated time delay. The result is a class of filters with a range of applications including abstract sound synthesis, decorrelation, steganography, impulse response measurement, and audio effects processing.

In section 2, we show how an arbitrary group delay trajectory can be used to drive the impulse response of an allpass filter. Section 3 introduces an equalization method to give the allpass filter a near constant amplitude envelope. Section 4 discusses details on the implementation of these filters and section 5 presents some applications. Finally, section 6 concludes the paper and suggests areas of future work.

2. METHOD

2.1. Allpass Filters

In discrete time, allpass filters are realized as having poles within the unit circle and zeros that are reciprocally reflected outside the unit circle at the same angle. A first-order allpass filter can be described by the transfer function

$$G(z) = \frac{-\rho + z^{-1}}{1 - \rho z^{-1}}\,, \tag{1}$$

and the difference equation

$$y(t) = -\rho x(t) + x(t-1) + \rho y(t-1)\,, \tag{2}$$

where ρ is the position of the pole. Its impulse response is therefore

$$g(t) = \begin{cases} 0, & t < 0 \\ -\rho, & t = 0 \\ (1-\rho)^2 \rho^{t-1}, & t \geq 1 \end{cases}\,, \tag{3}$$

Its magnitude is by definition

$$\left| G(e^{-j\omega}) \right| = 1\,, \tag{4}$$

and it has the phase response

$$\sphericalangle G(e^{-j\omega}) = -\arctan \frac{(1-\rho)^2 \sin(\omega)}{(1-\rho)^2 \cos(\omega) - 2\rho}\,. \tag{5}$$

The group delay of a filter is defined as the negative derivative of phase with respect to frequency,

$$\tau(\omega) = -\frac{d\phi(\omega)}{d\omega}\,. \tag{6}$$

The first order allpass filter has the group delay

$$-\frac{d\phi(\omega)}{d\omega} = \frac{1-\rho^2}{1+\rho^2 - 2\rho\cos(\omega)}\,. \tag{7}$$

Given a group delay trajectory, we can calculate the phase,

$$\phi(\omega) = -\int_0^\omega \tau(\omega) d\omega\,, \tag{8}$$

and since an allpass filter has unit magnitude, the time domain impulse response is simply

$$g(t) = \mathcal{F}^{-1}\left[e^{j\phi(\omega)}\right]\,. \tag{9}$$

It is important to note that cascading multiple allpass filters, even with different values for ρ, still creates an allpass filter.

2.2. Choosing a Group Delay Characteristic

If the group delay is set to a constant value,

$$\tau(\omega) = k\,, \tag{10}$$

(9) produces a band limited impulse. A linear chirp results if the group delay changes linearly, traversing a fixed frequency bandwidth in each time step,

$$\tau(\omega) = \eta\,\omega\,. \tag{11}$$

If the group delay trajectory traverses each octave in the same length of time, an exponential chirp is the result,

$$\tau(\omega) = \eta\ln(\omega)\,. \tag{12}$$

The observation here is that the group delay trajectory can be viewed as a function that maps the frequency axis to a set of time delays. Moreover, we can set this group delay trajectory to be any arbitrary function which determines this time/frequency relationship, so long as each frequency corresponds to one time delay value. Depending on the group delay trajectory, the resulting filters can be used as a type of novel audio effect. In some cases, the impulse responses themselves have interesting sounds and could stand on their own as a new synthesis method.

We will now discuss some of the myriad ways to set the group delay trajectory to produce musical effects and sounds.

2.3. Stair-Stepped Group Delays

We can discretize the linear sweep from above into n ascending segments, spaced by m in time with

$$\tau(\omega) = \frac{\lfloor n\,\omega \rfloor}{m}\,, \tag{13}$$

where ω is in normalized frequency ($\omega \in [0, 1]$). This discretized chirp can effectively be viewed as passing an impulse through a set of n bandpass filters that are each delayed by a multiple of m in time. For an example, see Fig. 1.

Since this stair-step equation has discontinuities introduced by the floor function, it might be desirable to suppress the time/frequency leakage by smoothing out the discontinuities. This can be done, for example, with the hyperbolic tangent function. The following expression,

$$\tau(\omega) = m\left(\frac{\tanh\left(\frac{\omega}{\alpha n} - \frac{1}{\alpha}\left\lfloor\frac{\omega}{n}\right\rfloor - \frac{2}{\alpha}\right)}{2\tanh\left(\frac{2}{\alpha}\right)} + \frac{1}{2} + \left\lfloor\frac{\omega}{n}\right\rfloor\right)\,, \tag{14}$$

produces a smoothed staircase group delay where n determines the number of segments and α is a smoothing parameter. When α is close to 0, (14) produces an output similar to (13). As α increases, the output of (14) becomes smoother and closer to the continuous chirp from (11). Fig. 2 shows an example of a smoothed stair-step group delay filter.

We can warp the frequency scale to control the frequency-dependent energy contained in each bandpassed-segment,

$$\tau(\omega) = \frac{\lfloor n\,\nu\{\omega\} \rfloor}{m}\,, \tag{15}$$

where $\nu\{\cdot\}$ is a function that determines the frequency axis warping, for example $\nu\{\omega\} = \omega^{1/2}$ would cause the higher frequency segments have a larger bandwidth.

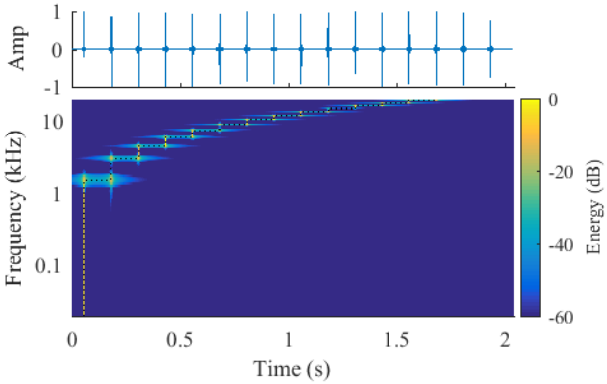

Figure 1: A sixteen-segment stair-stepped filter. The group delay is plotted on top of the spectrogram with a dotted black line like done by [29].

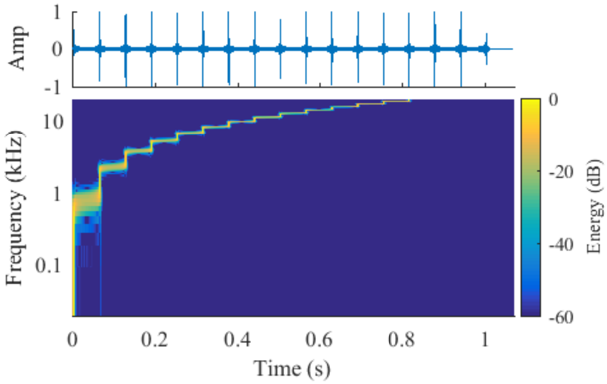

Figure 2: A smoothed sixteen-segment stair-stepped filter, with $\alpha = 0.05$.

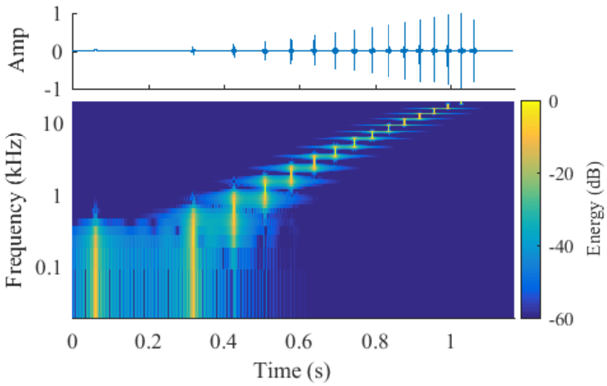

Figure 3: A sixteen-segment stair-stepped filter with time and frequency warping.

We could also warp the time scale for when these segments appear with the function $\zeta\{\cdot\}$

$$\tau(\omega) = \frac{\zeta\{\lfloor n\,\omega \rfloor\}}{m}\,. \tag{16}$$

For example, $\zeta\{x\} = x^2$ would compress the time interval between the early pulses and spread the later ones out in time.

We can naturally combine both time and frequency warping into the same expression,

$$\tau(\omega) = \frac{\zeta\{\lfloor n\,\nu\{\omega\} \rfloor\}}{m}\,. \tag{17}$$

Fig. 3 shows a filter with a group delay chosen to have even energy per octave and to compress the time interval in the high frequencies.

In (13), (15), (16), and (17), it may be beneficial to normalize the numerator to the interval $[0, 1]$ so the factor m does need to be modified to compensate for global timing changes introduced by time and frequency warping.

The group delay function can also be chosen to scramble the order of the frequency segments. For example,

$$\tau(\omega) = \begin{cases} d_1, & \omega \in [0, \omega_1) \\ d_2, & \omega \in [\omega_1, \omega_2) \\ \vdots & \vdots \\ d_N, & \omega \in [\omega_{N-1}, \omega_N] \end{cases}\,, \tag{18}$$

where $\{d_1, d_2, \cdots, d_N\}$ are the delays for each frequency region. An example of this "arpeggiated" group delay filter can be seen in Fig. 4. If the delay times are allowed to repeat one can create "chordal" structures, as seen in Fig. 5.

2.4. Modulated Group Delay

In addition to setting the group delay to create a "stair-step" function like described in 2.3, the group delay can also be modulated. For example, a group delay such as

$$\tau(\omega) = k\cos(2\pi\omega f + \phi)\,, \tag{19}$$

where k determines the total length of the filter, f the frequency of the modulator, and ϕ the initial phase, would create a filter that oscillates—or "chirps"—up and down simultaneously in different frequency bands f times across the audio band. When f is very small (below about 5), the individual chirp trajectories are audible (see Fig. 6). When f is between about $(5, 100)$, the filter sounds like a modulated signal (see Fig. 7). Above this modulator speed, the energy starts to pile up at discrete points in time, likely associated with the extrema of modulation, and the filter sounds like a sequence of clicks or "echoes" (see Fig. 8).

We can, again, warp the frequency axis to adjust how many oscillations occur within a certain frequency region. For example,

$$\tau(\omega) = k\cos(2\pi\,\nu\{\omega\}f + \phi)\,, \quad \nu\{\omega\} = \omega^{1/2} \tag{20}$$

would have an equal number of oscillations in each octave. Now, a large modulator f no longer stacks energy at discrete time points, but rather creates other perceptual chirp trajectories, as seen in Figs. 9 and 10.

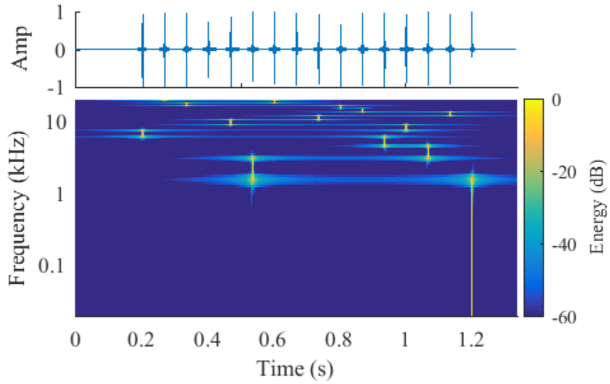

Figure 4: A sixteen-segment "arpeggiated" (scrambled) stair-stepped filter.

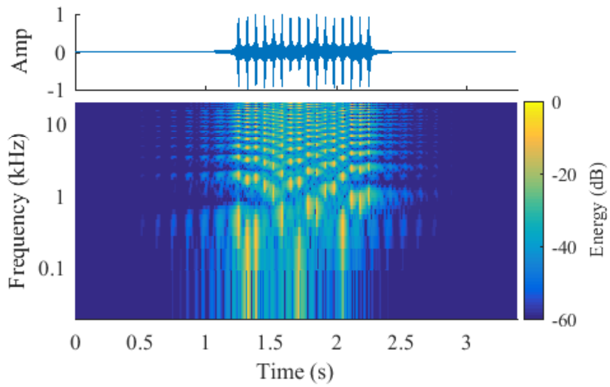

Figure 5: A sixteen-segment "chordal" (multiple frequencies at the same time) stair-stepped filter.

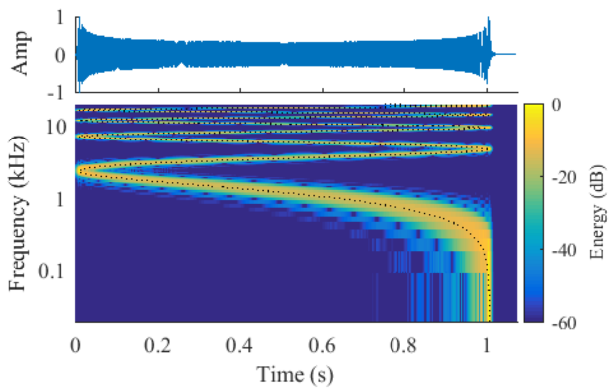

Figure 6: A slow (5 Hz) sine-modulated filter. The group delay is plotted on top of the spectrogram with a dotted black line.

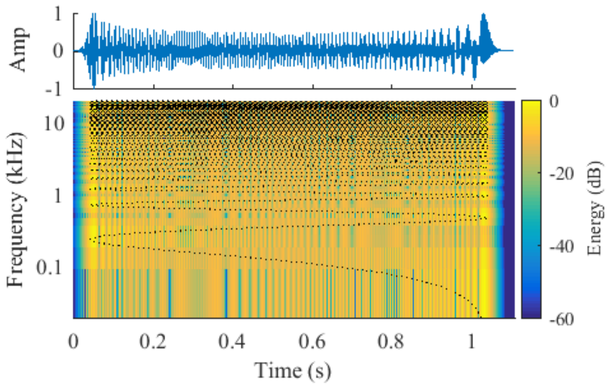

Figure 7: A medium (50 Hz) sine-modulated filter. The group delay is plotted on top of the spectrogram with a dotted black line.

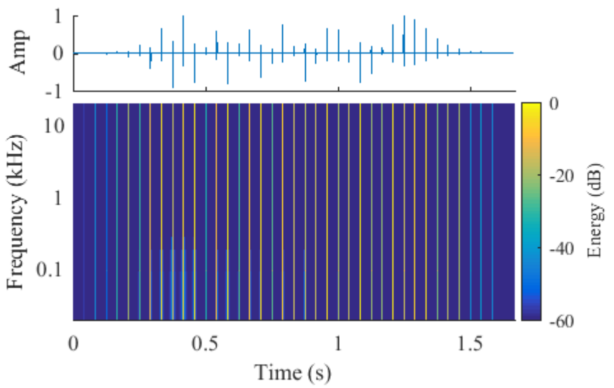

Figure 8: A fast (1 kHz) sine-modulated filter.

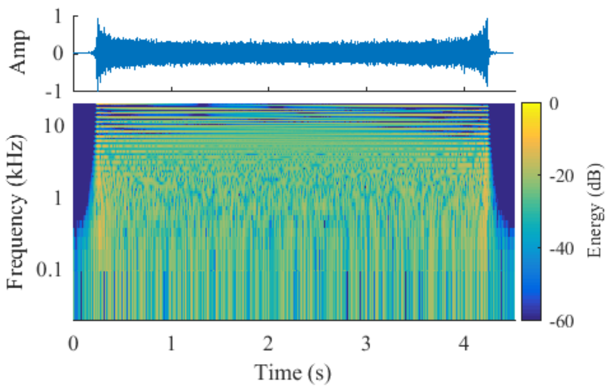

Figure 9: A sine-modulated filter with warped frequency axis.

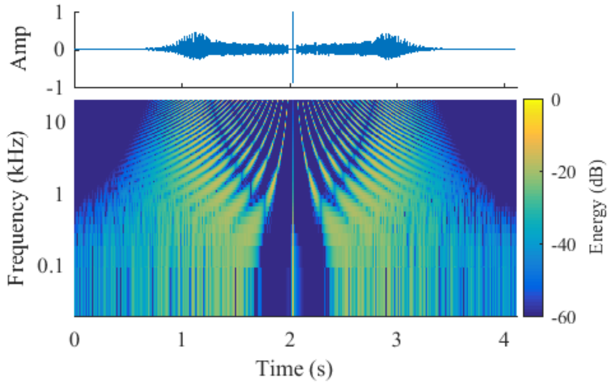

Figure 10: A sine-modulated filter with warped time and frequency axes.

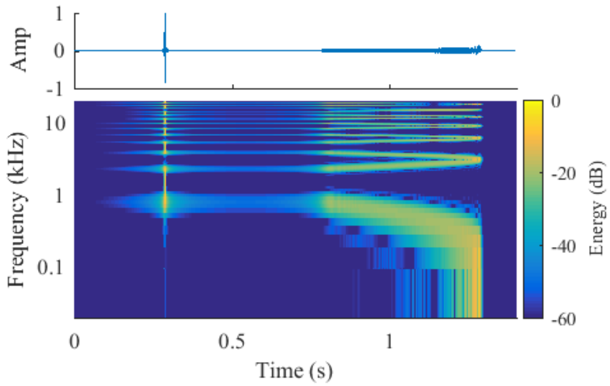

Figure 11: A filter created with a sinusoidally modulalated group delay that is soft-clipped on one side.

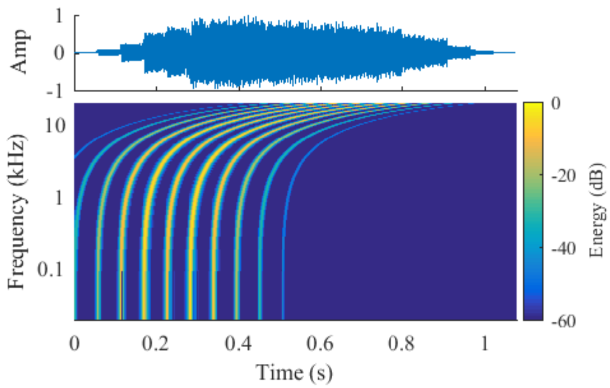

Figure 12: A "spring"-like group delay created by a fast, sinusoidal modulated group delay which is multiplied by an exponential ramp.

To (19), we can further add amplitude modulation with the modulator $m_a(\omega)$

$$\tau(\omega) = k\cos(2\pi\omega f + \phi)\, m_a(\omega)\,, \tag{21}$$

or frequency modulation with the modulating signal $m_f(\omega)$,

$$\tau(\omega) = k\cos(2\pi\omega f + m_f(\omega))\,. \tag{22}$$

Both of these methods can create interesting sounds with complex spectra. Naturally, the modulating function used as the group delay trajectory need not be a sinusoidal signal and there are many other possibilities. For example, Fig. 11 shows a modulated group delay filter with saturating distortion and Fig. 12 shows a sine modulated group delay with an additional exponential modulation used to create a spring reverb-like effect.

2.5. Probabilistic Group Delay

Another way to "draw" the group delay trajectory is probabilistically. If $\tau(\omega)$ is randomly drawn from a Gaussian distribution, the resulting impulse response will simply be a burst of enveloped noise with a duration proportional to the width of the Gaussian. We can also construct the group delay as a frequency dependent "drunk-walk" path.

Let there be N maximum-delay waypoints, each defined at some frequency. One such method would be to define the waypoints according to a perceptual criteria, like one waypoint per ERB-band center frequency. By smoothly interpolating between these waypoints, we define a frequency-dependent "area" within which we will draw our group delay curve.

To generate the actual group delay trajectories, we divide the maximum delay/frequency curve into β segments, distributed according to some function of frequency, $\zeta\{\omega\}$. If $\zeta\{\omega\}$ is linear, more segments will be in the high frequencies. If $\zeta\{\omega\}$ is an ERB warping, the segments will be approximately evenly distributed across the range of human hearing.

For each of these segments, we randomly choose a delay that falls within the maximum delay for that frequency. These segments are then either aligned along their leading or trailing edge (e.g., each segment can be between $[0, \texttt{maxdelay}]$, or centered about their midpoints $(-\texttt{maxdelay}/2, \texttt{maxdelay}/2)$. This now defines a set of discrete frequencies where the group delay is set. To define a continuous group delay function, we simply interpolate this set of points.

When β is small (see Fig. 13), there will be relatively few segments and the resulting filter may "sound chirpy," and when β is large (see Fig. 14), the result will sound more like enveloped noise. In between these extremes, filters designed like this can sound "metallic," like what is shown in Figs. 15 and 16. In all cases, the maximum frequency/delay curve defines an area that will be filled by the β segments. By defining these filters with a random process, we can generate a large number of mutually decorrelated allpass filters that have the same type of sound.

2.6. Hand-Drawn Group Delay

There are naturally many ways to design the group delay. One method which allows flexibility is to simply draw it by hand. Using a grid where the user sets a delay for each quantized frequency (potentially on a warped frequency axis), and then smoothly interpolating between the grid points and potentially resampling and scaling in time is one such method. Additionally, one could draw

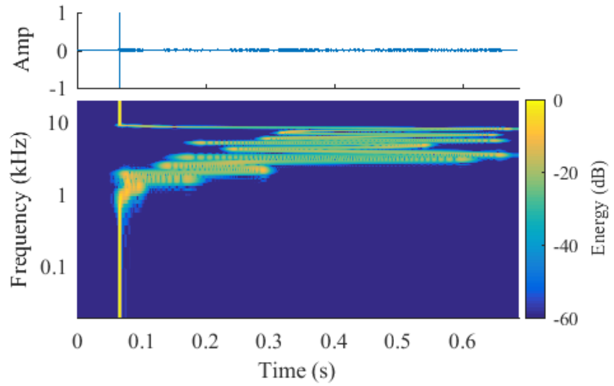

Figure 13: A probabilistic allpass filter with $\beta = 75$ and the segments aligned at $t = 0$. Note that a large frequency region was selected to have constant group delay so the filter only effects a narrow bandwidth.

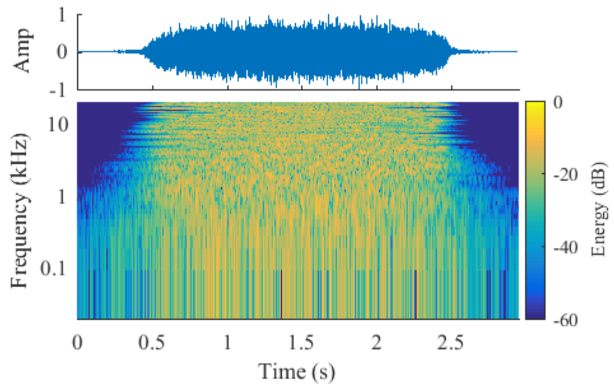

Figure 14: A probabilistic allpass filter with $\beta = 2000$ and the segments centered.

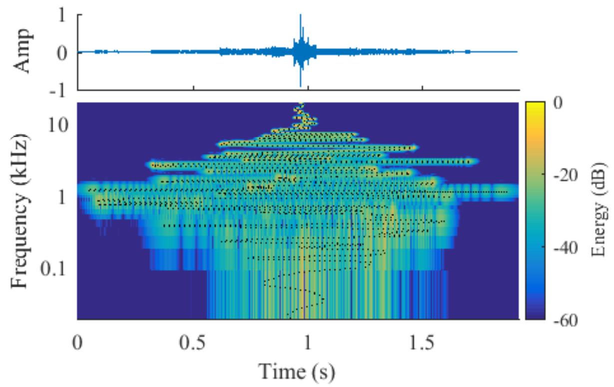

Figure 15: A probabilistic allpass filter with $\beta = 100$ and the segments centered. The group delay is plotted on top of the spectrogram with a dotted black line.

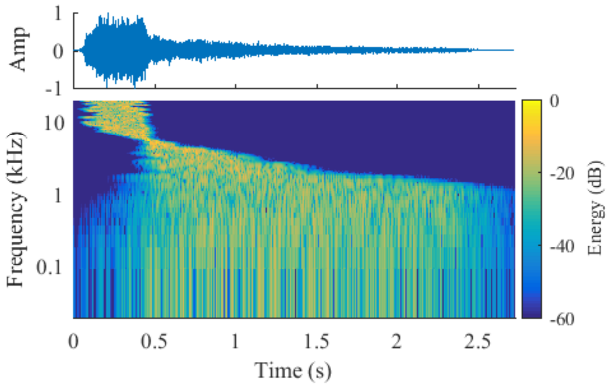

Figure 16: A probabilistic allpass filter with $\beta = 1000$ and the segments aligned at $t = 0$. Note that some of the time/frequency waypoints were set to have negative values.

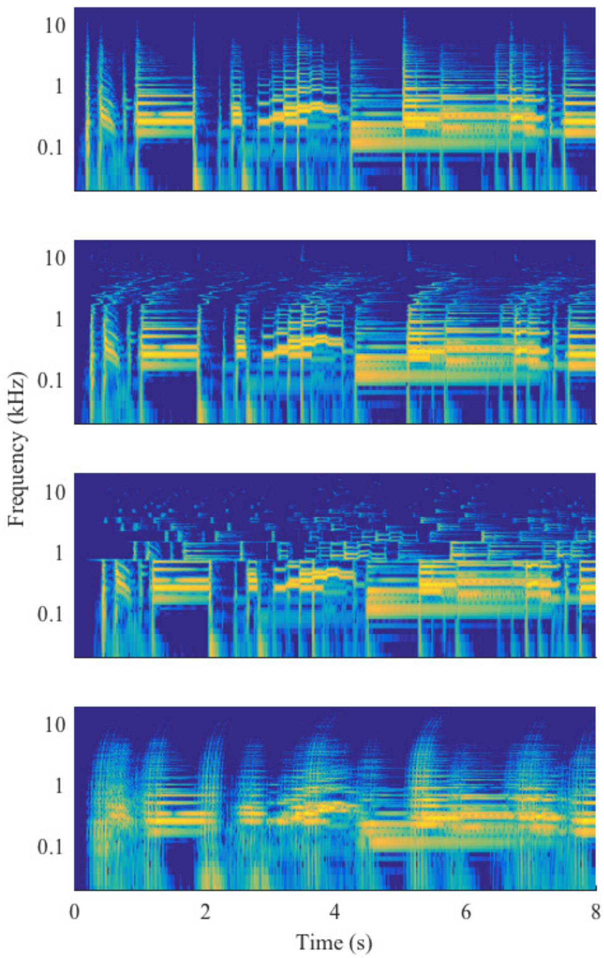

Figure 17: A guitar track unprocessed (top) and processed through the allpass filter shown in Fig. 13 (second), Fig. 4 (third), and Fig. 12 (bottom). Note how the various filters affect the timing of the spectral components of the guitar.

the maximum delay curve described in section 2.5 and then statistically generate the group delay.

2.7. Processing

Not only can these filters have interesting sounding impulse responses, they can provide the basis for interesting audio processes. For example, Fig. 17 shows the spectrogram of a guitar track processed through allpass filters such as the ones shown in Figs. 13, 4 and 12. In the first case, one hears the "tonal" components of the guitar accompanied by "chorus" of high-frequency chirps resulting from time-smeared transients. The second filter creates an "arpeggiated" sound. The last filter adds a spring reverb-like sound.

3. EQUALIZATION

In these filters, energy is conserved since these filters are allpass. However, the slope of the group delay determines the amount of time over which each frequency region is spread. In regions where frequencies are more spread in time, the instantaneous amplitude will be relatively lower than regions where the frequencies are less dispersed. In many cases, one would want the allpass filter that results from the methods above, but sometimes it is desirable to design a signal with a constant amplitude envelope. Our perception of the amplitude envelope of a signal is associated with a temporal time constant. Equalizing the amplitude envelope could help prevent unintentional changes in level.

Given a group delay characteristic, it is possible to calculate the amplitude envelope as a function of frequency, as shown in [29]. Denote by $\omega_\pm$ two close frequencies with difference Δ and mean ω,

$$\Delta = \omega_+ - \omega_- \qquad\qquad \omega = \frac{(\omega_- + \omega_+)}{2}\,. \tag{23}$$

An allpass filter will have Δ/π energy in the interval $[\omega_-, \omega_+]$. This energy is roughly equal to the signal energy in the time interval $[\tau(\omega_-), \tau(\omega_+)]$,

$$\frac{\Delta}{\pi} \approx |\tau(\omega_-) - \tau(\omega_+)|\,\frac{a^2(\omega)}{2}\,. \tag{24}$$

Taking the limit $\Delta \to 0$ and solving for the amplitude envelope a yields

$$a(\omega) = \left[\frac{\pi}{2}\left|\frac{d\tau(\omega)}{d\omega}\right|\right]^{-\frac{1}{2}}. \tag{25}$$

By approximating the inverse of (25), we have a good equalization filter $u(\omega)$ that yields a near constant crest factor

$$|u(\omega)| \approx \frac{1}{a(\omega)} = \left[\frac{\pi}{2}\left|\frac{d\tau(\omega)}{d\omega}\right|\right]^{\frac{1}{2}}. \tag{26}$$

Naturally, this equalized filter will change the amplitude relationships across frequency and brings out (by amplifying) the frequency regions with slowly changing group delays.

4. IMPLEMENTATION AND COMPLEXITY

These filters can be computationally expensive as the implementation of these filters with complex group delay characteristics require a large number of filter sections. A single biquad allpass filter adds a cumulative 2π phase. Short filters without a large amount of integrated group delay could be implemented with the methods described by [32,33] or [34], however some of the filters described in this paper are temporally so long or have such a significant amount of integrated group delay that it would be impractical to implement them in the time domain. Moreover, a time-domain implementation would add a significant amount of pure delay due to the large number of filters in cascade. Instead, we implement these filters by finding their impulse responses with (8) and (9). We typically pre-compute the impulse responses offline and apply the filters to input in real-time with a fast convolution algorithm [35–37].

Since we are using the Discrete Fourier Transform (DFT) to find the impulse response related to a specific phase characteristic, it is necessary to use a DFT long enough to implement the filter. Since the group delay trajectory tells us how much each frequency is delayed, we simply need a DFT length longer than the maximum delay. If the DFT is not long enough, the specified delays will alias.

If the group delay changes slowly and smoothly, there will be little spectral leakage between the DFT bins. If the group delay changes quickly or there are large discontinuities between frequency indices of the sampled group delay, there is a higher likelihood of spectral leakage. This can be partially mitigated by using a longer DFT length. We typically use a DFT length that is twice as large as the maximum delay and trim the length of the resulting impulse response.

5. APPLICATIONS

The filters described above can be used in a variety of applications. When the total duration of the impulse response is long, these filters can be quite musical on their own. They can be, and have been, used as sound effects and musical components of electro-acoustic music [38, 39]. When the total IR duration is short, these filters can be useful for processing other sounds. For example, introducing chords and arpeggios to a piano, complex echoes and delays to drums, "birdies" to the transients of guitar strums, and inharmonic distortion to vocals. Some audio examples of these filters as sound effects and processors can be found online at `https://ccrma.stanford.edu/~kermit/website/gdapf.html`.

In addition to musical sounds, these filters have other practical uses. When the maximum group delay is shorter than about 30 ms, one does not necessarily perceive the frequency-dependent delays and the IR of the filter could sound like a click. If multiple, different filters are used together, these filters make effective decorrelators.

Like for decorrelation, if one generates many mutually decorrelated allpass filters, one could foreseeably use them for steganography, where a message or data is encoded with a set of filters that can only be decoded by correlating the code with the correct key.

These filter can also be used for impulse response measurement. While sine sweeps, Golay codes, and pseudo-random noise sequences are effective tools for probing systems, they are all unpleasant and aggressive sounds. One could use the filter design approach from this paper to create "musical" test signals that are less irritating to hear.

6. CONCLUSIONS AND FUTURE WORK

In this paper we have demonstrated a novel method for sound processing and synthesis which uses allpass filters formed by setting a group delay trajectory that sets frequency-dependent delays. By choosing the group delay characteristic, these filters can create a large variety of interesting sounds either on their own or for processing other sounds. These filters are passive and energy conserving, and are useful for abstract sound synthesis, audio effects processing, decorrelation, steganography, and impulse response measurement, among others.

We have shown several methods for constructing the group delay, including stair-stepped, modulated, probabilistic, and hand-drawn methods. We also showed a method for equalizing the allpass filter to have a near constant amplitude envelope. In addition to creating new sounds and effects, these filters can be used to produce new takes on classic audio effects.

The filters presented here have all been static. Moving forward, we would like to find an efficient method for implementing these filters that can accommodate time-varying designs.

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