EXPERIMENTAL STUDY OF GUITAR PICKUP NONLINEARITY

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ABSTRACT

In this paper, we focus on studying nonlinear behavior of the pickup of an electric guitar and on its modeling. The approach is purely experimental, based on physical assumptions and attempts to find a nonlinear model that, with few parameters, would be able to predict the nonlinear behavior of the pickup. In our experimental setup a piece of string is attached to a shaker and vibrates perpendicularly to the pickup in frequency range between 60 Hz and 400 Hz. The oscillations are controlled by a linearizion feedback to create a purely sinusoidal steady state movement of the string. In the first step, harmonic distortions of three different magnetic pickups (a single-coil, a humbucker, and a rail-pickup) are compared to check if they provide different distortions. In the second step, a static nonlinearity of Paiva's model is estimated from experimental signals. In the last step, the pickup nonlinearities are compared and an empirical model that fits well all three pickups is proposed.

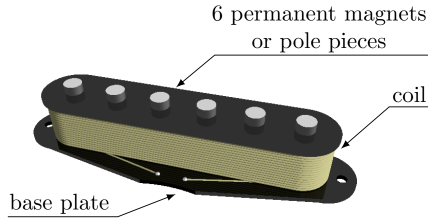


Figure 1: *Schematic representation of a "single-coil" pickup.*

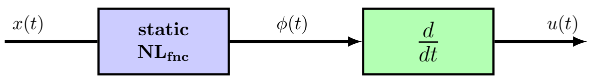


Figure 2: *Block diagram of the pickup nonlinear model, with $x(t)$ the string displacement, $\phi(t)$ the magnetic flux, and $u(t)$ the output voltage of the pickup.*

1. INTRODUCTION

The beautiful sounds created by musical instruments, whether acoustic or electro-acoustic, relies very often on a nonlinear mechanism and the electric guitar is obviously no exception. The heart of an electric guitar is a pickup, a nonlinear sensor that captures the string vibrations and translates them into an electric signal [1, 2, 3, 4]. A magnetic pickup is basically composed of a set of permanent magnets surrounded by an electric coil (see Figure 1). A ferromagnetic string vibrating in the vicinity of the pickup results in a variation of the magnetic flux through the coil, and, according to the Faraday's law, an electrical voltage is then induced across the coil [3].

Since first pickups appeared almost a century ago, there have been thousands of pickup models, each of them providing different output. Almost all the electric-guitar players have probably asked the puzzling question of what distinguishes one particular pickup from another. Why is it that some sound warmer, some cleaner and some more distorted ? The answer to this question is important not only for guitar players but also for pickup manufactures and for digital audio effects engineers, especially those working with instrument synthesis [5, 6, 7]. A few models of pickup available in the literature may help to find the answer to this tricky question.

Some of these models are based on physical approaches using either integral equations [3, 8] or port-Hamiltonian systems [9, 10] while others are based on block-oriented models combining linear and nonlinear blocks together [11, 12]. In [11] Paiva shows that the sound of a pickup is influenced by three main properties: 1) the pickup position and width which are closely related to the string vibration, 2) the pickup high impedance which together with the input impedance of the device to which the guitar is plugged forms a linear filter, and 3) a nonlinear behavior of the pickup. The main core of the Paiva's model [11] describing the magneto-electric conversion is based on a simple static nonlinearity representing the nonlinear relation between the string displacement and the magnetic flux, followed by a time derivative (see Figure 2). In [13, 14] we have experimentally shown, that this simple model, called Hammerstein system, is sufficiently precise for pickup nonlinear modeling and that more complicated models, such as a Generalized Hammerstein model, converge back to the simple Hammerstein system.

This paper focuses on experimental measurement of the static nonlinear block of Paiva's model and on comparison of nonlinear behavior of several pickups. Three different pickups of brand Seymour Duncan are chosen: 1) "SSL-5" - a single-coil pickup, 2) "SH-2N" - a humbucker (double-coil) pickup and 3) "STHR-1B Hot Rails" - a humbucker rail pickup using a rail in place of a row of six pole pieces. After presenting our experimental setup in section 2, a preliminary comparative measurement of harmonic distortion of each pickup is presented for two different pickup/string distances (section 2.1). Even if distortions of these three pickups are of the same nature, the difference in distortion between each pickup is visible not only in the spectra but also in the time-domain waveforms of the voltage output. To find the origin of this difference, we focus on the experimental identification of the static nonlinearity of the pickup. In section 3, the pickup nonlinear behavior is characterized experimentally leading to estimation of the input-output curve representing the static nonlinearity of the pickup. Finally, in section 4, all tested pickups are compared and an empirical law that fits well all the measurements at different pickup/string distances is proposed.

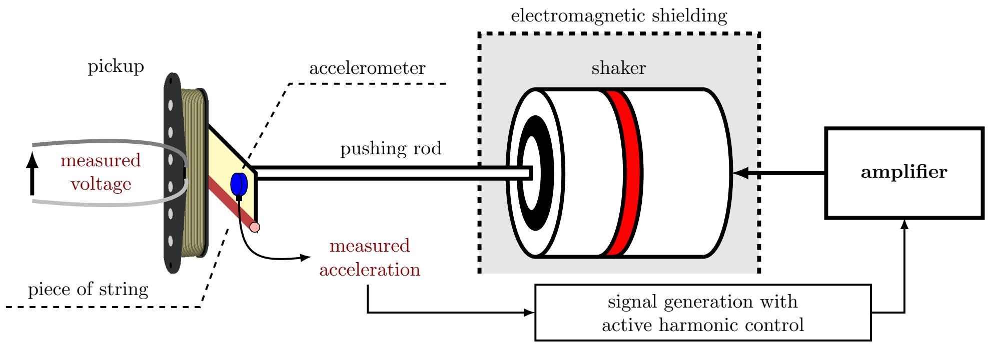


Figure 3: *Configuration of the measurement setup.*

2. EXPERIMENTAL SETUP & HARMONIC DISTORTION TEST

On the one hand, the identification of linear and nonlinear systems is generally based on a knowledge of input and output signals where the input signal is perfectly controlled (usually a random or deterministic signal such as sine, swept-sine, multi-tone, ...). On the other hand, the input signal of the pickup is the string displacement, guided by the laws of vibrations, which is difficult to control.

In order to overcome this problem, a specific measurement setup depicted in Figure 3 is used. A piece of string (8 cm long and 1 mm in diameter) is glued to a composite plate (3 x 8 cm) which is rigidly connected to an electrodynamic shaker (Brüel & KjæLDS V406). The shaker, driven by a Devialet D-premier amplifier, R.M.E Fireface 400 sound card, and a personal computer, is used as a source of the string displacement. The string is then placed next to the pickup's 6th pole piece (low E string position) at a distance d_0 so that the string can oscillate around d_0 with amplitude $\pm d_{max}$ (Figure 4). To avoid a possible disturbance by the electromagnetic field of the shaker, an electromagnetic shielding cage is placed around the shaker. An accelerometer PCB 352C22 is fixed to the composite plate (firmly fixed to the string). The sound card R.M.E Fireface 400 is then used to acquire both the signal $a(t)$ from the accelerometer (through a PCB sensor signal conditioner 482C series) and the output voltage $u(t)$ from the pickup (directly connected to the sound card instrument input with input impedance of 470 kΩ).

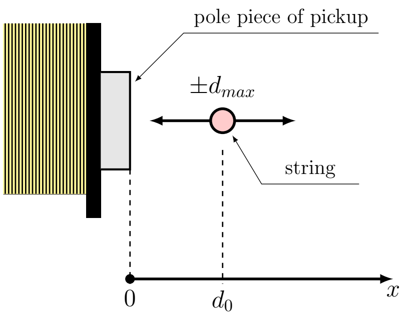


Figure 4: *Schematic representation of the pickup/string distance d_0 given by the distance between the string rest position and the pickup magnet (or its pole piece), and of the amplitude d_{max} of the string excursion, defining the total displacement $d_0 \pm d_{max}$.*

2.1. Measurement of pickup's harmonic distortion

Such a measurement setup can be used to control the input signal (displacement of the string - deduced from the measured acceleration) and to analyze the behavior of the pickup. However, in order to analyze the pickup from the nonlinear point of view, one would desire that the shaker, used to displace the string, behaves linearly. Otherwise, the displacement would be contaminated by the nonlinearities of the shaker which would make the identification of the nonlinear behavior of the pickup much more difficult. In [13, 14], a procedure based on swept-sine measurement [15], that allows to post-process the measured data and to identify the nonlinear system under test in terms of Generalized Hammerstein model, has been used. While efficient, this technique cannot fix the excitation signal in real time.

Recently, a simple and robust adaptive technique that can predistort the input signal in a real time to create a perfect periodical signal at the output of the shaker (with spectral purity up to 100 dB) has been proposed in [16]. Using this technique, we can generate a pure harmonic displacement even for large amplitudes, canceling completely the nonlinearity of the shaker. Therefore, if the measured acceleration, and consequently the string displacement, is ensured to be purely harmonic, the harmonic distortion observed at the output voltage of the pickup can be associated only to the nonlinearity of the pickup.

The results of this "harmonic excitation" experiment are depicted in Figure 5 for all three tested pickups and for two different pickup/string distances $d_0 = 3$ and 5 mm. The harmonic excitation with $d_{max} = 2$ mm and frequency 80 Hz, chosen in accordance with free vibrations of an E-string, is imposed to the string. One can see from Figure 5 that the nonlinear distortion of all three tested pickups has a similar character, but each output differs. The difference is visible not only in the frequency domain but also in the time domain. It is of no surprise that both humbucker pickups (SH-2N and the "STHR-1B Hot Rails") provide signals with higher level and higher distortion compared to a single-coil "SSL-5". The humbucker pickups also exhibit much stronger distortion when placed closer to the string ($d_0 = 3$ mm). When placed further from the string ($d_0 = 5$ cm), the level and distortion are surprisingly much more similar. In order to understand the origin of these differences in distortion generated by the pickups, we provide the following set of experiments, all of them conducted using the experimental setup described at the beginning of this section.

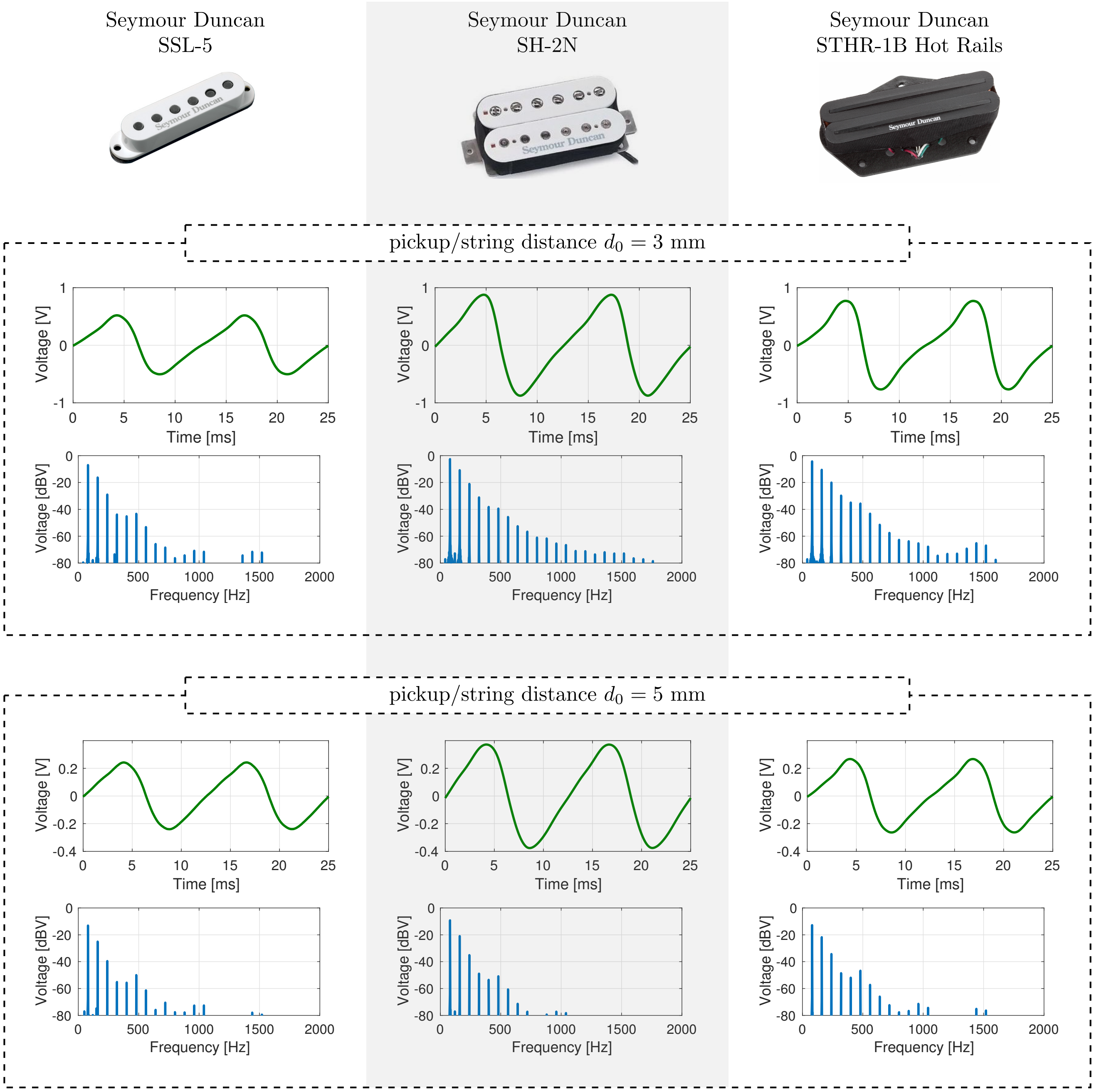


Figure 5: *Waveforms (green) and spectra (blue) of the output voltage of three pickups under test: a single-coil pickup (SSL-5) on the left, a humbucker double-coil pickup (SH-2N) in the middle (with gray background), and a rail pickup (STHR-1B Hot Rails) on the right. Results depicted for two different pickup/string distances $d_0 = 3$ mm and $d_0 = 5$ mm. The string is placed in front of the pickup's 6th pole piece (low E string position).*

3. EXPERIMENTAL ESTIMATION OF THE PICKUP NONLINEARITY

Since the Paiva's block-model [11] (static nonlinearity followed by the time derivative, see Figure 6(a)) has been experimentally verified in [13, 14], following experiments are focused on identification of the static nonlinearity directly from experimental signals.

To make a link between the physics and the block-model from Figure 6(a), we recall the Faraday's law of induction that defines the voltage $u(t)$ generated at the output of a coil with N turns as

$$u(t) = -N\frac{d\Phi_c(t)}{dt}, \tag{1}$$

$\Phi_c(t)$ being the magnetic flux passing through the coil. Comparing this law with the block-model, we can see, that the signal $\Phi(t)$ (time integral of the voltage $u(t)$) has the dimensions of the magnetic flux [V · s] and differs from the real flux $\Phi_c(t)$ of the coil by sign and by number of turns N. The signal $\Phi(t)$ can be easily deduced from the measured voltage $u(t)$ simply by integrating

$$\Phi(t) = \int_{-\infty}^{t} u(t')dt' + C. \tag{2}$$

Note, that an unknown constant of integration C, inherent in the construction of anti-derivatives, appears at the end of Equation (2). This constant is related to the direct component (DC) of the magnetic flux passing through the voice coil.

The block-model then assumes that there exists a direct static nonlinear relation between the time integral of the voltage $\Phi(t)$ and the string displacement $x(t)$

$$\Phi(t) = \mathrm{NL_{fnc}}\Big\{x(t)\Big\}. \tag{3}$$

In the following, these two quantities $\Phi(t)$ and $x(t)$, derived from the measured voltage and acceleration[1], are used to estimate the input-output (I/O) relation of the static function $\mathrm{NL_{fnc}}$. The term "static" means, that it is independent of frequency, and that an I/O relation depicted in a graph (translated to Matlab language as `plot(x,phi)`) should exhibit no area inside the closed curve.

In Figure 6, the procedure explained above is depicted using the experimental results ("SSL-5" pickup, $d_0 = 3$ mm, and $d_{max} = 2.8$ mm). Since the I/O relation should not depend on frequency, we use 60 Hz (resonant frequency of the shaker) to achieve larger amplitude d_{max} of displacement. The validation of this assumption through an experiment is provided in section 3.2. From the results depicted in Figure 6 we can conclude that the signal $\Phi(t)$, depicted in Figure 6(c), is very distorted and that, contrary to the assumption of a static nonlinearity, the I/O relation (Figure 6(d)) gives a curve with a non-negligible area inside the closed curve. The area can be simply explained by a small phase shift due to a time delay between the measured voltage and acceleration (e.g. due to the sensor signal conditioner). After applying a time delay (the actual time delay of 0.23 ms is the same for all measurements and independent of frequency), the I/O relation (Figure 6(e)) shows a nice smooth curve (with no closed curve area) that represents an estimate of the static nonlinear function $\mathrm{NL_{fnc}}$.

As shown in the experiment with the harmonic distortion presented in Figure 5 the distortion differs for different pickup/string positions d_0. The following two experiments are thus focused on: 1) the influence of the pickup/string position d_0 on the static nonlinear function $\mathrm{NL_{fnc}}$, and 2) the frequency dependence.

[1] integrating and differentiating in the frequency domain

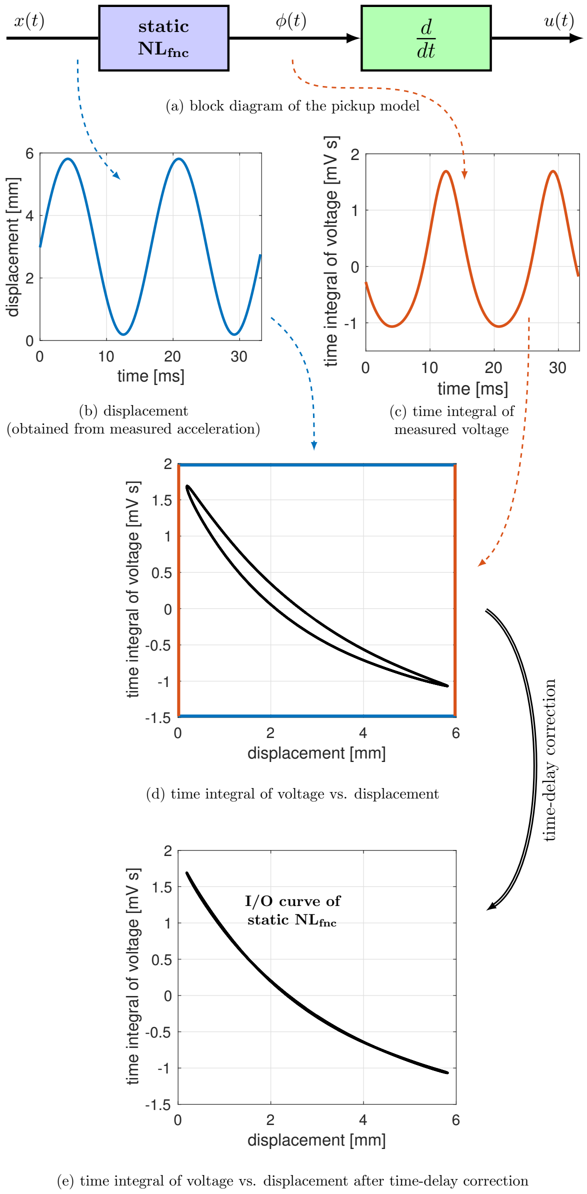


Figure 6: *(a) A block diagram of the pickup model with the measured signals (b) string displacement (obtained from the measured acceleration), (c) time integral of the measured voltage, and (d-e) a plot of time integral of voltage vs. string displacement in an I/O graph to estimated the form of the static nonlinear function; (d) without any correction, (e) a time-delay compensation is applied. Measurements performed on a SSL-5 pickup with a string placed at $d_0 = 3$ mm from the pickup's 6th pole piece (low E string position) and oscillating harmonically with amplitude $d_{max} = 2.8$ mm at 60 Hz.*

3.1. Influence of the pickup/string distance d_0

The previous experimental result shows that the I/O relation of the static function $\mathrm{NL_{fnc}}$ representing the static nonlinear block can be obtained directly from the string displacement (derived from measured acceleration) and from the time integral of the measured voltage of the pickup's output. The physical model of the guitar pickup proposed in [8] suggests that the nonlinear behavior of the pickup is influenced by the pickup/string distance d_0.

To study the influence of d_0 on the static nonlinear function $\mathrm{NL_{fnc}}$ of the model (see Figure 2), the following experiment is made for pickup/string distances d_0 = 3 mm, 5 mm, 7 mm, and 10 mm. The amplitude of the string displacement is $d_{max} = 3.5$ mm for all the measurements except for $d_0 = 3$ mm, for which d_{max} is set to 2.8 mm to avoid the string hitting the pickup.

The resulting I/O curves representing the $\mathrm{NL_{fnc}}$ are shown in Figures 7(a-d). The nonlinear I/O relation between the string displacement and the time integral of the voltage is much more nonlinear when the string is closer to the pickup (e.g. for $d_0 = 3$ mm, see Figure 7(a)) than when the string is much further ($d_0 = 10$ mm, see Figure 7(d)) even if, in this particular case with d_0 = 3 mm, the amplitude d_{max} of the string displacement is larger for the measurement made at other pickup/string distances.

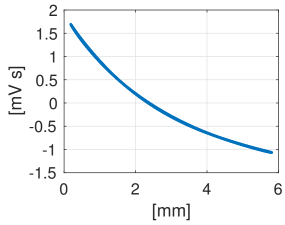
We can see from Figures 7(a-d) that the nonlinear behavior of the pickup (I/O curve) varies a lot with the pickup/string distance d_0. One could consequently conclude that when a guitar player changes the distance d_0 of the string, a different static nonlinear function $\mathrm{NL_{fnc}}$ applies. Indeed, as shown in Equation (2), the signal $\Phi(t)$, obtained as a time integral of the measured voltage $u(t)$, is missing the unknown constant of integration C. Consequently, the I/O curves can be offset (shifted vertically) with the same relative result. The offset has no consequence on the output voltage $u(t)$ due to the time derivative block. Figure 7(d) shows each I/O curve from Figures 7(a-d) plotted with an offset to achieve the best superposition. The superposition of the I/O curves, measured at different pickup/string distances d_0, is almost perfect, indicating that there is only one static nonlinear function $\mathrm{NL_{fnc}}$ no matter the pickup/string distance d_0. Therefore, when a guitar player changes the distance d_0 of the string, the same static nonlinear function $\mathrm{NL_{fnc}}$ applies.

3.2. Independent of frequency ?

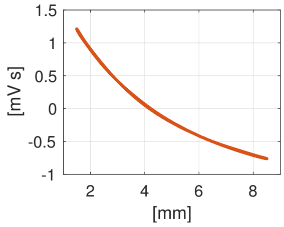
To verify that the nonlinear function $\mathrm{NL_{fnc}}$ is really static, i.e. independent of frequency, a similar measurement is conducted on the SSL-5 pickup for different frequencies (60 Hz, 80 Hz, and 400 Hz) for a given pickup/string distance $d_0 = 4$ mm. The amplitude d_{max} differs for each measurement due to the physical limits of the shaker at different frequencies. While at low frequencies (e.g. 60 Hz) the shaker can provide a d_{max} close to 3 mm, for the same driving voltage at 400 Hz it provides a d_{max} lower than 1 mm.

Each I/O relation estimated from the measured signals for different frequencies is provided in Figure 8. The I/O curve obtained for 60 Hz (Figure 8(a)) is more curved than the one obtained for 80 Hz (Figure 8(b)), indicating a higher nonlinear distortion at 60 Hz. The I/O curve for 400 Hz (Figure 8(c)) is almost a perfect straight (linear) line.

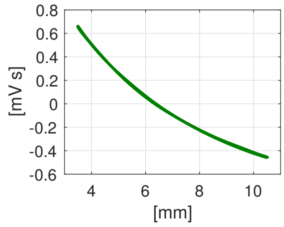
Similarly to the previous results, one could conclude that the pickup behavior vary a lot with frequency (high distortion for low frequencies and low distortion for high frequencies). Nevertheless, it must not be forgotten that the string displacement have not the same value of amplitude d_{max} (see the x-axis in Figures 8(a-c)).



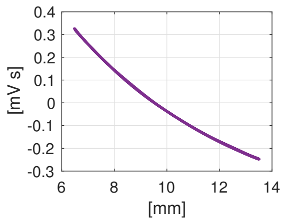
(a) $d_0 = 3$ mm



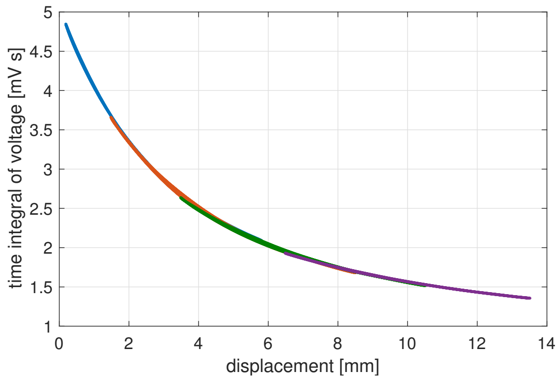
(b) $d_0 = 5$ mm



(c) $d_0 = 7$ mm



(d) $d_0 = 10$ mm



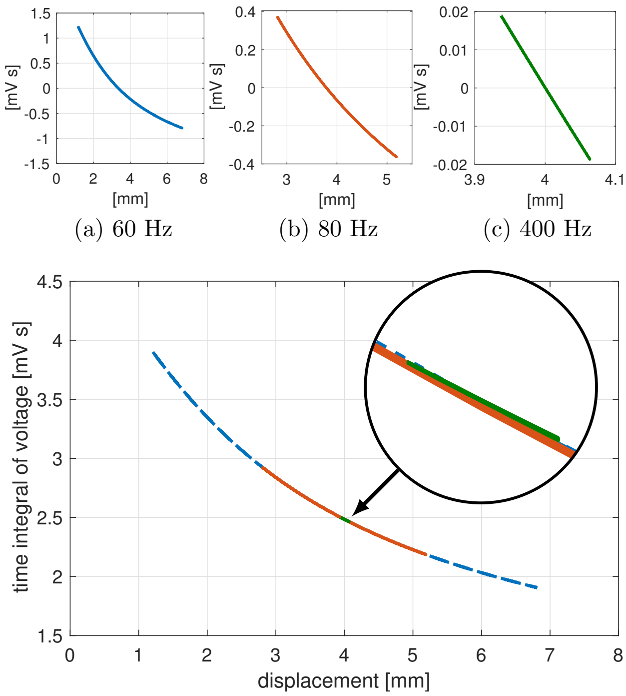
(e) superposition of measurements with $d_0 = 3$ mm, 5 mm, 7 mm, and 10 mm

Figure 7: *I/O graphs (time integral of measured voltage vs. string displacement) for four different pickup/string distances d_0 (a) 3 mm, (b) 5 mm, (c) 7 mm, and (d) 10mm. All the four I/O graphs are superposed in (e) where each time integral of measured voltage is offset by an unknown constant of integration. Measurements performed on a SSL-5 pickup with a string oscillating harmonically with amplitude $d_{max} = 2.8$ mm around $d_0 = 3$ mm and with amplitude $d_{max} = 3.5$ mm around $d_0 = 5$ mm, $d_0 = 7$ mm, and $d_0 = 10$ mm. The frequency is chosen to be $60\ Hz$ in order to maximize the displacement of the shaker. The string is placed in front of the pickup's 6th pole piece (low E string position).*

As in the previous experiment, each of the I/O curves can be offset (shifted vertically) to compensate for the unknown constant of integration (Equation (2)). The superposition of the offset I/O curves measured at different frequencies, depicted in Figure 8(d), results in an almost perfect superposition confirming the hypothesis of non frequency dependent static nonlinear function $\mathrm{NL_{fnc}}$.

3.3. Discussion

These experiments conducted on the SSL-5 pickup show that a single static nonlinear function $\mathrm{NL_{fnc}}$ can be used, no matter the distance d_0 to which the guitar player sets the string. In other



(a) 60 Hz (b) 80 Hz (c) 400 Hz

(d) superposition of 60 Hz, 80 Hz, and 400 Hz curves

Figure 8: *I/O graphs (plot of the time integral of measured voltage vs. string displacement) for three different excitation frequencies (a) 60 Hz, (b) 80 Hz, (c) 400 Hz, and (d) all the three I/O graphs for three different frequencies superposed in one I/O graph. Each time integral of measured voltage is offset by an unknown constant of integration. Measurements performed on a SSL-5 pickup with a string placed at $d_0 = 4$ mm in front of the pickup's 6th pole piece (low E string position).*

words, d_0 and d_{max} join together into a single variable x representing the instantaneous distance of the string from the pickup (see the schematic representation in Figure 4). Consequently, the parameters d_0, d_{max}, and the frequency of string vibration, are parameters associated to the string displacement (input of $\mathrm{NL_{fnc}}$), not to the pickup nonlinearity $\mathrm{NL_{fnc}}$ or its parameters.

Note also that the output voltage of the pickup is proportionally related to the gradient of the I/O curve, or, i.e. to the gradient of the magnetic flux (time integral of the voltage). Indeed combining Equations (1 - 3), one can write

$$u(t) = \frac{d\Phi(t)}{dt} = \frac{d\mathrm{NL_{fnc}}\left\{x(t)\right\}}{dt} = \frac{\partial \mathrm{NL_{fnc}}}{\partial x}\frac{dx(t)}{dt}. \quad (4)$$

The output voltage is thus proportional both to the velocity of the string and to the instantaneous gradient of the static nonlinear function $\mathrm{NL_{fnc}}$. It is thus straightforward to guess from Figure 7 how the string position d_0 influences the level of output signal and the nonlinear distortion. For string position $d_0 = 3$ mm, close to the pickup, the I/O curve is very steep indicating high induced voltage at the output of the pickup. It is also curved due to the gradient variation, indicating a high nonlinear distortion. In the opposite way, for larger distance from the pickup (e.g. $d_0 = 10$ mm) the slope is weak and the curve flatter, resulting in a smaller voltage output with less distortion.

4. A SINGLE EMPIRICAL MODEL FOR ALL PICKUPS

In this section we provide the comparative results of three different pickups of brand Seymour Duncan: "SSL-5" - a single-coil pickup, "SH-2N" - a humbucker (double-coil) pickup, and "STHR-1B Hot Rails" a humbucker rail pickup. The goals of this section are threefold: (1) to verify that the findings proposed in the previous experiment on SSL-5 pickup also apply to the other pickups, (2) to see if there is any difference between the nonlinear I/O curves of each pickup and, if yes, what makes this difference, and (3) to provide an empirical model (other than the polynomial one) that, with minimal amount of parameters, would be able to predict the pickup nonlinear behavior.

4.1. Experiments on different pickups

All the three tested pickups are measured in the same way as the SSL-5 pickup in the previous section. The comparative table provided in Figure 9 shows the estimated $\mathrm{NL_{fnc}}$ of these three pickups. Observing the I/O graphs created by superposing the four measurements for different d_0 one can note that the conclusions proposed in the previous section for the single-coil (SSL-5) also apply to the humbucker (double-coil) pickups SH-2N and STHR-1B Hot Rails. Roughly speaking we can also predict the behavior of the pickups by observing the shapes of each nonlinear function $\mathrm{NL_{fnc}}$. Following Equation (4), in which the pickup output voltage is proportional to the instantaneous value of the gradient of the $\mathrm{NL_{fnc}}$, we can deduce that the SH-2N and STHR-1B will produce higher output level than the SSL-5 when the string is placed close (e.g. $d_0 = 3$ mm) to the pickup since the slope (gradient) of the $\mathrm{NL_{fnc}}$ is much higher. On the other hand when the string is placed at the distance of $d_0 = 5$ mm, the slopes of the $\mathrm{NL_{fnc}}$ are smaller and similar for all the three pickups, thus the amplitude of the output voltage should be smaller and similar for all pickups. This is perfectly correlated with the results presented in Figure 5 in which the signals and spectra of the pickups' output voltage are depicted. Similarly, the level of distortion of these signals is well correlated with the variations of the slope of the static nonlinear functions $\mathrm{NL_{fnc}}$ from Figure 9.

4.2. Single empirical model

To replicate the laws of physics that describe the nonlinear behavior of the pickup, represented by the static nonlinear function $\mathrm{NL_{fnc}}$, it is desirable to find a fitting function that would fit the I/O law of the $\mathrm{NL_{fnc}}$ using few parameters. It could be used not only for sound synthesis of an electric guitar but also to quantitatively differentiate pickups through these parameters.

A polynomial fit (based on Taylor series) is usually the most common way to model a static nonlinear function when the analytical formula is not known or simply to reduce the computational cost of a platform on which the model of the pickup is implemented [1, 12]. The main disadvantages of a polynomial fit are, first, a missing physical interpretation and, second, the need of a high number of parameters in order to fit the curve correctly. Note, that the spectra of the SH-2N voltage output measured for a sinusoidally oscillating string around $d_0 = 3$ mm with amplitude $d_{max} = 2$ mm (see Figure 5) contains more than 20 harmonics. The polynomial fit would thus need at least 20 coefficients to reproduce a similar result which would be very impractical. Reducing the number of coefficients would lead to lower precision of the model. Moreover, all the measurement I/O curves presented in this paper for different kinds of pickups rather exhibit a law similar to an exponential decay one which is difficult to fit a polynomial with few coefficients. Our attempt to fit the I/O curves with one exponential law revealed to be unsuccessful (high deviation on extremities of the I/O curves). A sum of two exponentials seemed to fit better the I/O behavior, but two exponentials require too many parameters for the fitting and do not seem to be really justified from the point of view of the physical laws of electromagnetism.

On the other hand, the basic magnetic field $B(x)$ of a cylindrical magnet (or a solenoid) along its x-axis is analytically described as [8]

$$B(x) = \frac{B_r}{2}\left(\frac{x+L}{\sqrt{r^2+(x+L)^2}} - \frac{x}{\sqrt{r^2+x^2}}\right), \quad (5)$$

where B_r is the remanent flux density of the magnet, r its radius, and L its length. Despite the fact that this relation is not describing the magnetic flux of the coil as a response to oscillating string in its proximity, we tried to find the best fit to the I/O curves using Equation (5) ... with no success (similar results that the exponential model). Inspired by this simple model, we tried to modify equation (5) empirically to find a better fit. Indeed, replacing both square roots by cube roots surprisingly led to very successful fit for all three studied pickups. Then, the following equation

$$\mathrm{NL_{fnc}}(x) = A\left(\frac{x+L_{eq}}{\sqrt[3]{r_{eq}^2+(x+L_{eq})^2}} - \frac{x}{\sqrt[3]{r_{eq}^2+x^2}}\right), \quad (6)$$

provides an empirical model of the static nonlinear function $\mathrm{NL_{fnc}}$ with only three parameters. Moreover, since the model is based on the physical basis, even if modified empirically, we can associate the model parameters to an equivalent cylindrical magnet with an equivalent radius r_{eq}, and an equivalent length L_{eq}. The constant A then includes the remanent flux density B_r as well as the string characteristics such as diameter and material properties.

The measured I/O curves of the static nonlinear function $\mathrm{NL_{fnc}}$ depicted in Figure 9 are fitted using Equation (6). The best fit is plotted in a black & white dashed curve in the same figure and the estimated parameters A, L_{eq}, and r_{eq} are provided for each pickup under each I/O curve. Note that the parameters L_{eq} and r_{eq} correspond to credible values of the length and radius of an equivalent magnet.

The output $\Phi(t)$ of the static nonlinear block (Figure 2) can be easily derived from Equation (6) as

$$\Phi(t) = A\left(\frac{x(t)+L_{eq}}{\sqrt[3]{r_{eq}^2+[x(t)+L_{eq}]^2}} - \frac{x(t)}{\sqrt[3]{r_{eq}^2+x^2(t)}}\right). \quad (7)$$

This equation can be used to directly calculate the output $\Phi(t)$ of the static nonlinear function to any string displacement $x(t)$, sinusoidal ($x(t) = d_0 + d_{max}\sin(2\pi f_0 t)$) or musical (offset by d_0). One can then simply calculate the time-derivative of $\Phi(t)$ to directly deduce the output voltage $u(t)$ of the pickup. Another possibility is to provide directly the voltage output $u(t)$ as a function of input string vibration $x(t)$ (still offset by d_0) using the gradient of the static nonlinear function $\mathrm{NL_{fnc}}$ (see Equation (4)) as

$$u(t) = \frac{\partial \mathrm{NL_{fnc}}}{\partial x}\frac{dx}{dt}, \quad (8)$$

with

$$\frac{\partial \mathrm{NL_{fnc}}}{\partial x} = A\left(\frac{(x(t)+L_{eq})^2+3r_{eq}^2}{3\left([x(t)+L_{eq}]^2+r_{eq}^2\right)^{4/3}} - \frac{x^2(t)+3r_{eq}^2}{3\left(x^2(t)+r_{eq}^2\right)^{4/3}}\right). \quad (9)$$

5. CONCLUSIONS

In this paper, the pickup nonlinear behavior is studied from an experimental point of view considering three different pickups: a single coil pickup, a humbucker, and a rail pickup. The experimental setup using a piece of string attached to a shaker, whose displacement is actively controlled to provide a spectrally pure (without distortion) sinusoidal excitation, shows that the output of each studied pickup differs and that the distance between the string and the pickup plays an important role in voltage distortion. It is next shown, that the model proposed by Paiva, consisting of a static nonlinear function followed by a time derivative, corresponds to the experiments and that the static nonlinear function is independent of frequency and follows the same rule no matter the pickup/string distance. Moreover, an empirical model describing the pickup nonlinear behavior is proposed.

Future works on this topic will focus on the measurements of string displacement along x and y axes, on comparison between the same types of pickups of different brands, on the dependence on the string properties (width, material, ...), as well as on analytical modeling that could justify (or find better) the empirical model proposed in this paper.

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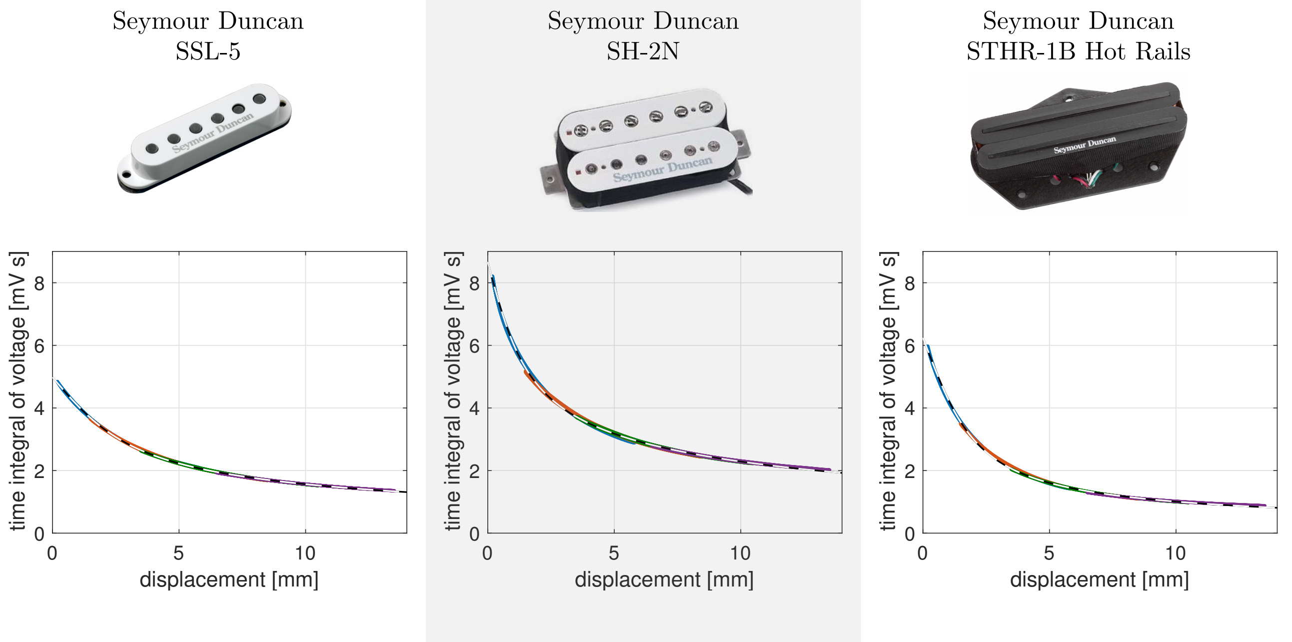

estimated parameters of the model from Equation (6)

SSL-5	SH-2N	STHR-1B Hot Rails
$A = 21.51 \cdot 10^{-3}$	$A = 40.67 \cdot 10^{-3}$	$A = 47.46 \cdot 10^{-3}$
$L_{eq} = 12.98$ mm	$L_{eq} = 9.86$ mm	$L_{eq} = 13.11$ mm
$r_{eq} = 2.77$ mm	$r_{eq} = 1.34$ mm	$r_{eq} = 1.88$ mm

Figure 9: *Plot of the static nonlinear functions (i.e time integral of voltage vs. string displacement depicted as an I/O graph) of three pickups under test: a single-coil pickup (SSL-5) on the left, a humbucker double-coil pickup (SH-2N) in the middle, and a rail pickup (STHR-1B Hot Rails) on the right. The data obtained from measurements are plotted in color (blue for $d_0 = 3$ mm, red for $d_0 = 5$ mm, green for $d_0 = 7$ mm, and violet for $d_0 = 10$ mm), and the fit using the empirical expression (6) is depicted in black & white dashed line. The string is placed in front of the pickup's 6th pole piece (low E string position).*

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