WATER BOTTLE SYNTHESIS WITH MODAL SIGNAL PROCESSING

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ABSTRACT

We present a method for accurately synthesizing the acoustic response of a water bottle using modal signal processing. We start with extensive measurements of two water bottles with considerations for how the level of water inside the bottles, the area covered by stickers attached to the exterior of the bottles, and the method of striking the bottles affect their sound. We perform modal analysis of these measurements and implement a real-time modal water bottle synthesizer.

1. INTRODUCTION

Previous work has examined the use of modal signal processing for synthesizing carillon bells [1, 2], artificial reverberation [3], cymbal synthesis [4], and more [5,6]. This work builds on the previous research to use modal synthesis for the accurate modelling of water bottle acoustics.

Although water bottles are not designed to function primarily as musical instruments, the authors have noticed that certain water bottles produce a pleasing resonant sound when struck with a mallet, knuckle, or other body part. The authors further noticed that water bottles can produce a great variety of sounds, depending not only on the shape and material of the bottle, but also on the amount of liquid contained within the bottle, as well as the amount and placement of stickers on the exterior of the bottle. Water bottle acoustics have not gone unnoticed by water bottle manufacturers, as at least one prominent manufacturer claims to be well aware of the pleasing acoustic properties of their bottles [7].

There exists considerable physics literature concerning the harmonic responses of various liquid containers and other thin-shell objects. Previous research has often focused on the modal behaviour of wine glasses and beer bottles, particularly with regard to the fundamental frequency at which the container vibrates [8–12]. Others focus particularly on the behavior of containers filled with heated beverages, such as coffee or hot chocolate [13–15]. [16] gives consideration to how liquid is distributed within a glass, while [17] examines the circular capillary waves on the surface of the liquid of a resonating wine glass. Finally, the authors of [18] explored the sympathetic resonance generated in a wine glass coupled to a vibrating string. Perhaps most closely related to the present research, [19] considers the musical possibilities of an iced tea can.

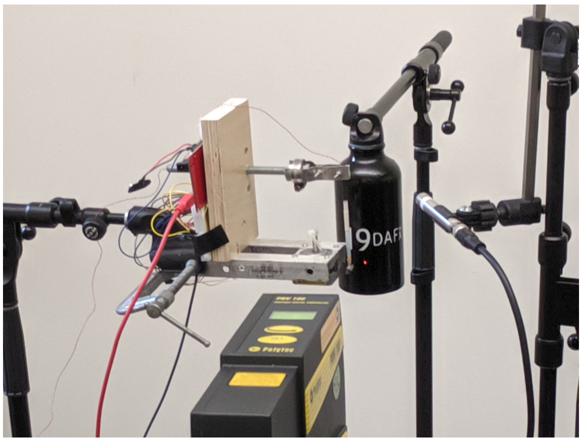

Figure 1: *Measurement setup for the DAFx water bottle*

We take measurements from a 32 oz. Wide Mouth HydroFlask,[1] compared to the water bottle given to attendees of the 2019 DAFx conference (seen in fig. 1), measured containing different amounts of water and with different placements of stickers on the exteriors of the bottles.

The structure of the paper is as follows: in §2 we describe our water bottle acoustical measurement procedure. §3 contains modal analysis of the water bottle measurements. Finally, in §4, we discuss our results and the implementation of a full water bottle synthesizer.

2. MEASUREMENTS

We wish to study the vibrational modes of the water bottle independent of the method by which it is struck. To do this, we strike the water bottle with a force sensing hammer and measure the surface velocity of the water bottle at a corresponding point using a laser Doppler vibrometer. We additionally capture the near field radiation using a pressure microphone. From both these measurements, we deconvolve the impact of the force hammer out of the signal. The full measurement setup can be seen in fig. 1. We also used a scanning vibrometer to observe the mode shapes and gain further insight into the vibrational characteristics of the bottles.

We made measurements of both a 32 oz. Wide Mouth HydroFlask and the complimentary DAFx19 water bottles with no water as well as $1/32$, $1/16$, $1/8$, $1/4$, $1/2$, and full with water. We additionally made measurements of the HydroFlask with the exterior covered in vinyl stickers as well as several intermediate and differently placed amounts of stickers.

Finally, we also wanted some understanding of different impacts on the water bottles. To make these measurements, we ad-

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[1] `https://www.hydroflask.com/32-oz-wide-mouth/`

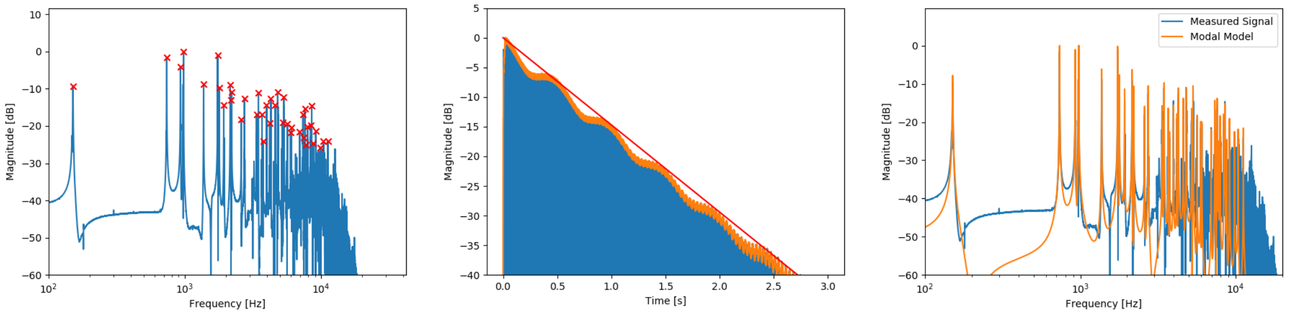

Figure 2: *Modal analysis pipeline: (left) picking the mode frequencies, (center) estimating the decay rate of a single mode, (right) using a least-squares fit to estimate the complex amplitudes of the modes that ideally resynthesize the original signal.*

hered an accelerometer to the inside of the HydroFlask and struck the outside (at the location of the accelerometer) with a variety of drum mallets and body parts.

3. ANALYSIS

3.1. Modal Analysis

Similar to the carillon bells modelled in [1, 2], we can use modal analysis to model the water bottle sounds as a sum of exponentially decaying sinusoids,

$$y(t) = \sum_{m=1}^{M} \alpha_m e^{j\omega_m t} e^{-t/\tau_m}\ , \tag{1}$$

where α_m is the complex amplitude, ω_m is the mode frequency, and τ_m is the decay rate for each mode m.

For modal analysis we use the helper functions provided by the `Python` audio signal processing library `audio_dspy`.[2] This process involves the following steps:

1. Picking the modal frequencies from the original recording.
2. Estimating the decay rate of each mode.
3. Estimating the complex amplitude of each mode.

The steps of the process are shown in full in fig. 2.

For finding the mode frequencies, we use a simple peak-picking algorithm over the Fourier Transform of the original signal.

For estimating the mode decay rates, we begin by filtering the signal using a 4th order Butterworth bandpass filter centered on the mode frequency, with a bandwidth of 30 Hz. We then apply a Root-Mean-Squared level detector as defined in [20] to estimate the energy envelope of the mode. Finally, we use a linear regression to estimate the slope of the energy envelope (measured in decibels per sample).

After computing the mode frequencies and decay rate, we perform a least squares fit to estimate the complex amplitude of each mode that most accurately resynthesizes the original recording.

3.2. Modal Resynthesis

For synthesizing the modes we use the Max Matthews phasor filter, as introduced in [21]. This filter is described by the difference equation:

$$y_m[n] = \alpha_m x[n] + e^{j\omega_m} e^{-1/\tau_m} y_m[n-1] \tag{2}$$

where τ_m is the mode decay rate described above, α_m is the complex amplitude of the mode, and ω_m is the mode frequency. This filter structure is known for having favorable numerical properties, as well as for being stable under real-time audio-rate parameter modulation. While the output of the filter is complex, a real signal can be constructed by taking either the real or imaginary part of the output signal (the two parts will be identical aside from a quarter-cycle phase shift). The results of the resynthesis process can be seen in fig. 3.

3.3. Water Level Analysis

Next we examine how the modal response of the water bottle changes as the water level in the bottle is varied.

3.3.1. Frequency Variation

Measurements of the HydroFlask bottle show that as the water level increases, the first mode frequency decreases, while the higher modes stay at the same frequency (see fig. 4). This suggests that the higher frequency modes are associated with vibration similar to that of a cylindrical shell, while the lowest frequency resonance is more similar to that of a beam clamped at one end with an additional mass at the end [22]. Since only the lowest mode changes frequency when the water level is changed, it is not apparent that the air column has an audible effect. This would likely change if the water bottle were uncapped. Figure 5 shows vibrometer scans of the HydroFlask filled roughly 40% with water as well as empty for three different mode shapes. The scans support the model hypothesis as the frequency of two higher shell modes are similar, while the frequency of the lowest beam-like mode decreases in frequency with added mass.

We can model the dependence between the frequency of the first mode and the water level using a modified sigmoid function of the form:

$$f(x) = C\left(1 - \frac{1}{1 + \exp(-b(x-a))}\right) + d \tag{3}$$

A comparison of the model with the measured data can be seen in fig. 6.

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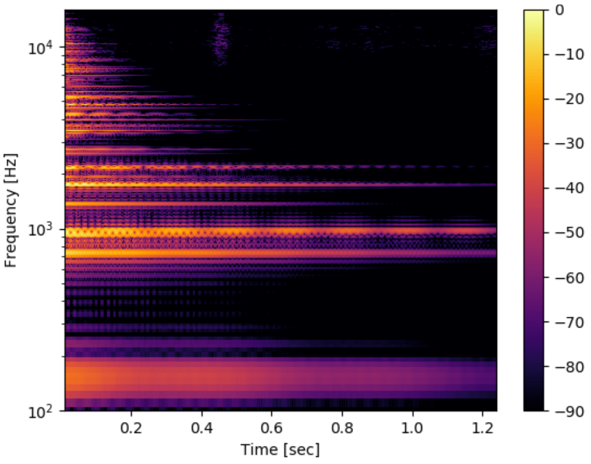
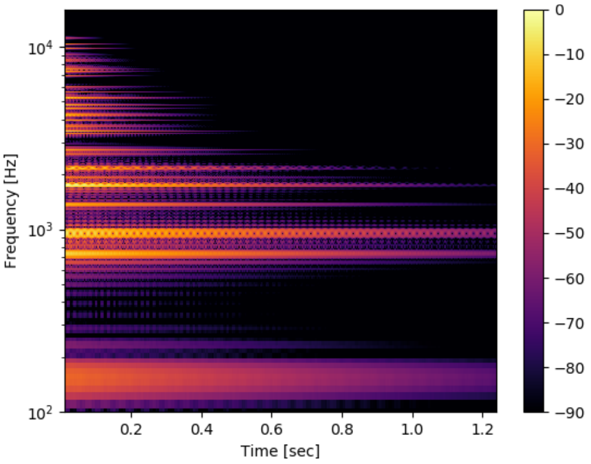
[2] `https://github.com/jatinchowdhury18/audio_dspy`



(a) Recorded



(b) Resynthesized

Figure 3: *Spectrograms of the full HydroFlask recorded (above) and resynthesized (below).*

3.3.2. *Damping Variation*

Further analysis shows that the damping of the lowest two modes varies with water level as well (see fig. 4). We can similarly model the variation of the mode decay rates with water level, using a 5th order polynomial to fit the decay rates of the first two modes (see fig. 7). The water level has very little impact on the damping of the higher modes. This also supports the hypothesis presented in §3.3.1 that the lowest modes are beam-like modes while the higher modes are cylindrical shell-like modes, as the mass loading disproportionately affects the beam-like modes.

3.4. Sticker Analysis

Initially, we compared two 32 oz. HydroFlask water bottles, one with stickers, one without, and noted that they had markedly different timbres. We then proceeded to take measurements of the bottle covered in varying amount of (removable) vinyl stickers. We found that the mode frequencies remained mostly unchanged with the addition of stickers, however, the mode dampings had noticeable variations (see fig. 8). Much like a “Moongel” damper pad on a drum head, a small application of stickers dramatically increases the damping. Adding a larger amount of stickers naturally increases the damping, although it has a less pronounced effect than the difference between no stickers and some stickers.

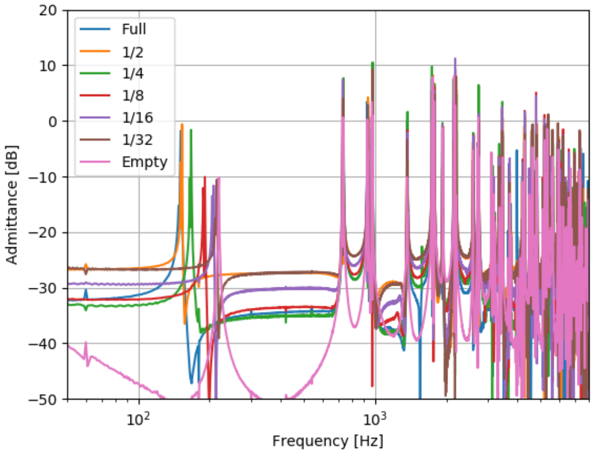
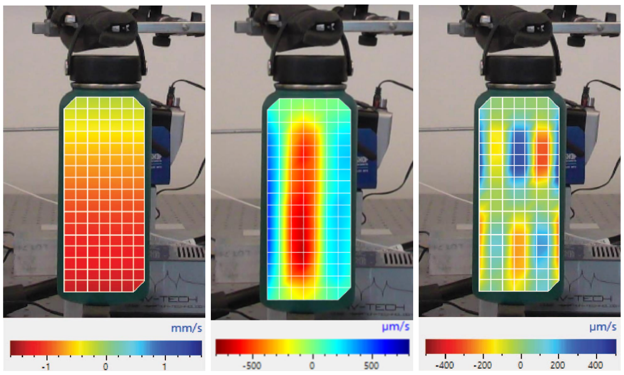
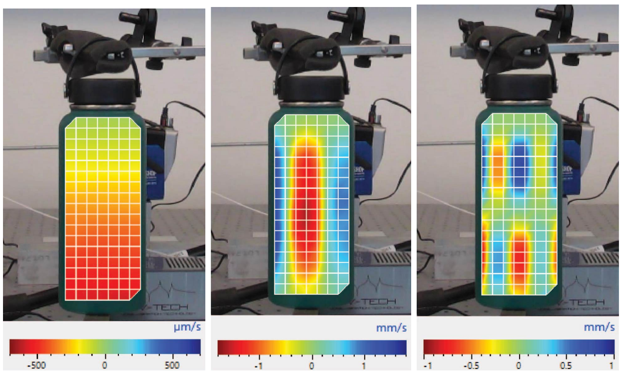


Figure 4: *Admittance of the HydroFlask with different amounts of water.*



(a) 157.5 Hz (b) 731.6 Hz (c) 1780 Hz



(d) 219.4 Hz (e) 731.3 Hz (f) 1780 Hz

Figure 5: *Vibrometer scans with roughly 40 % water (top) and without water (bottom).*

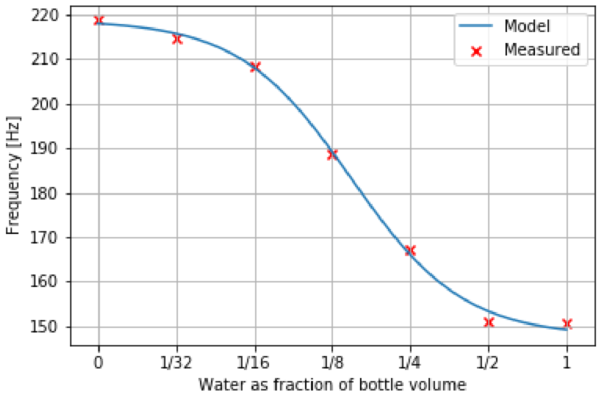


Figure 6: *Variation of the first mode frequency of the HydroFlask with the amount of water in the bottle*

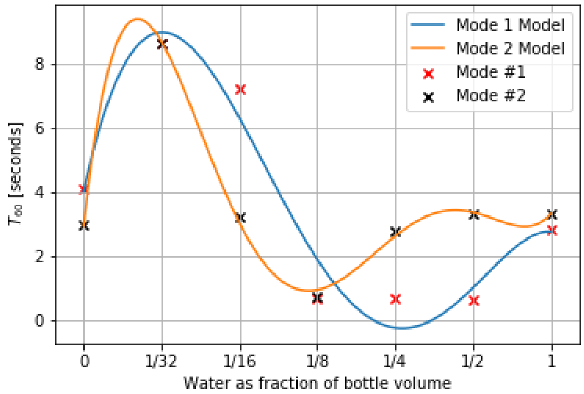


Figure 7: *Variation of the first two modes decay rates with the amount of water in the HydroFlask*

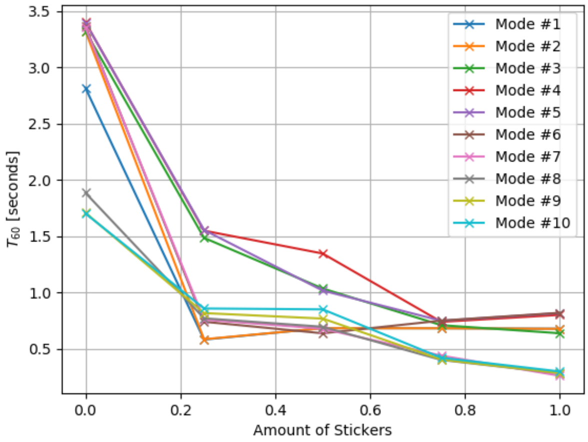


Figure 8: *Variation of the first ten modes decay rates with the amount of stickers on the HydroFlask*

3.5. Swinging Vibrato

When a water bottle is struck in such a way to produce an acoustic response, it often swings back and forth a little bit. This swinging causes the water within the bottle to move with the swinging, thereby causing some of the lower mode frequencies to oscillate. This oscillation manifests itself perceptually as a sort of vibrato effect which we refer to as “swinging vibrato.” We attempted to measure this swinging vibrato using the measurement setup described in §2, however the movement of the bottle caused the laser vibrometer to go out of focus, resulting in unsuitably noisy measurements. In lieu of usable measurements, we develop a physics-based model for swinging vibrato using the following steps:

1. Measure (or estimate) the height of the bottle.
2. Calculate the swinging frequency of the bottle.
3. Synthesize an initial amplitude and damping factor for the swinging oscillations.

Figure 9 shows a synthesized response of the HydroFlask, $1/16$th full with water, with exaggerated swinging vibrato applied to the first 5 modes..

3.5.1. *Bottle Height*

In cases where the height of the water bottle cannot be measured directly, it is possible to estimate the height from the bottle’s modal characteristics. For a typical cylindrical water bottle, the second lowest mode frequency corresponds to the bottle’s resonance along it’s vertical length (see figs. 5b and 5e). As such, the bottle height can be estimated as:

$$L = \frac{v_{sound}}{2f_2} \tag{4}$$

where $v_{sound} = 343$ m/s is the speed of sound in air, f_2 is the second lowest mode frequency, and L is the height of the bottle measured in meters. For the 32 Oz. HydroFlask, the second mode frequency $f_2 \approx 730$ Hz, corresponding to a bottle height $L \approx 23.5$ cm, which matches with the measured bottle height.

3.5.2. *Swinging Frequency*

The frequency of a pendulum can be derived from Newton’s Laws as:

$$f_{swing} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \tag{5}$$

where $g = 9.8\ \mathrm{m/s^2}$ is the acceleration due to gravity at the surface of Earth. It should be noted that the water bottle with water inside is not an ideal pendulum, resulting in an oscillation that is not purely sinusoidal. However in the first approximation, using a sinusoidal oscillation is sufficient. Using eq. (4) the swinging frequency of the HydroFlask can be calculated as $f_{swing} \approx 1.03$ Hz, which matches with the measured frequency.

3.5.3. *Swinging Amplitude and Damping*

The amplitude of a water bottle’s swinging oscillations typically depends on how hard the bottle is struck. As such, the amplitude of the swinging vibrato should vary proportionally with the desired loudness of the synthesized water bottle strike, for example with the “velocity” of a MIDI note. The damping of a water bottle’s swinging can be highly dependent on the method by which the water bottle is anchored. As such, the damping factor is left for the reader to determine for their own specific use cases.

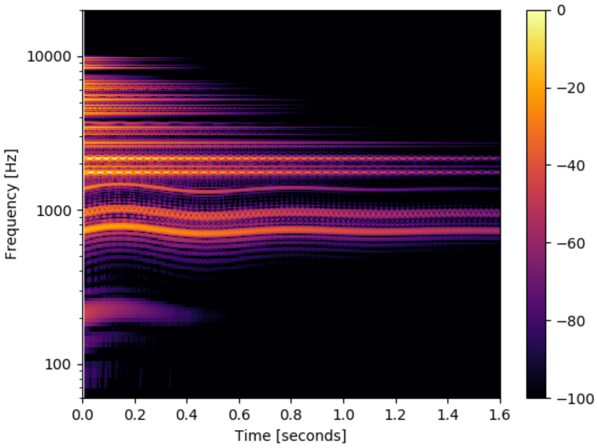


Figure 9: *A spectrogram of a synthesized waterbottle with exaggerated swinging vibrato on the lowest five modes.*

3.6. Impact Analysis

In real-world situations, a water bottle is typically struck using a body part, such as one's knees or knuckles, or using a striker such as a stick or mallet. With the goal of being able to synthesize these types of impacts, we used an accelerometer to measure the impacts of several body parts, as well as several types of drumsticks on a water bottle. The accelerometer was adhered to the inside of the water bottle corresponding to the location where the bottle was struck on the outside. Figure 10 shows the time and frequency domain measurements of the various impact types. Note that the apparent time delay present in the head and, to a lesser extent, the gong mallet impacts is due to their relatively soft exteriors. We can use any of these measurements as the input $x(t)$ in eq. (2) to synthesize the sound of a water bottle struck by the desired striker.

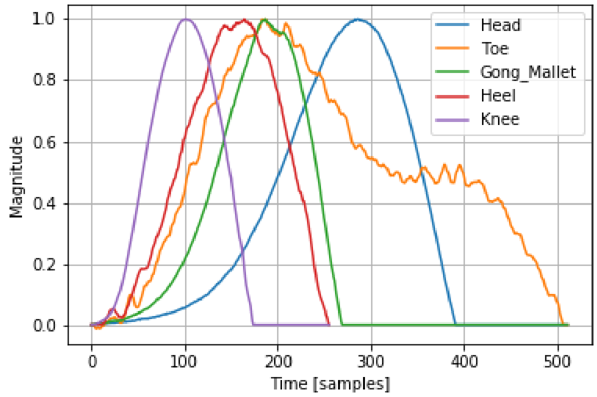
4. RESULTS

While one may expect the acoustic profile of a water bottle to be relatively simple, the measurements and analysis presented here demonstrate how significantly the sound changes when the water bottle is filled by different amounts of water and covered by different amounts of stickers. In an effort to demonstrate the musical possibilities of water bottle synthesis to a wider audience, we have developed two open-source audio plugins, described below. Source code for both plugins, as well as the modal analysis described in this paper is publicly available on GitHub.[3] Audio samples of various synthesized water bottles, several scanning vibrometer measurements, and video demos of the water bottle synthesizer plugin can be found online.[4]

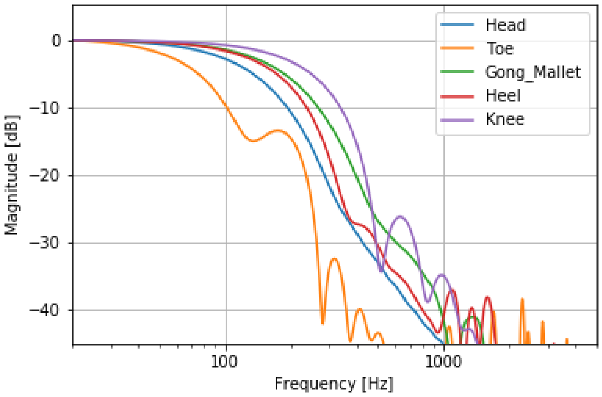
4.1. Water Bottle Synthesizers

4.1.1. HydroFlask Synthesizer

We have implemented a modal model of the 32 Oz. Wide Mouth HydroFlask as an 8-voice synthesizer plugin (VST/AU), using the JUCE/C++ framework (see fig. 11).[5] The synthesizer includes controls for the amount of water in the bottle, the number and placement of stickers on the bottle, and the option to strike the water bottle with a variety of objects.



(a) Time Domain



(b) Frequency Domain

Figure 10: *Time and frequency domain measurements of various impact drives.*

4.1.2. Bespoke Water Bottle Synthesizers

Water bottles are often cherished possessions—potentially partially due to the unique sound of a well-loved water bottle covered in memorable stickers and dents. In an effort to expand the world of water bottle synthesis to a wider audience, we have developed a system for creating a "bespoke" water bottle synthesizer for any water bottle. The first part of the system is comprised of a web app[6] containing the modal analysis algorithm defined in §3.1. Here users can upload their own water bottle recordings, and can download a `.waterbottle` file containing the modal characteristics of the corresponding recording. All water bottle characteristics are saved in a database. The second part of the system is comprised of an audio plugin that can recreate a modal water bottle synthesizer (including variable strikers and water amounts) from any `.waterbottle` file. Users can then download the modal

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[3] `https://github.com/jatinchowdhury18/modal-waterbottles`

[4] `https://ccrma.stanford.edu/~jatin/Waterbottles`

[5] `https://github.com/juce-framework/JUCE`

[6] `http://ccrmawaterbottles.pythonanywhere.com/`

Water 0.78
Knuckles
Swing Damp 0.70
Swing Modes 10

Figure 11: *Real-time modal synthesizer model of the HydroFlask 32 oz. Wide Mouth.*

characteristics of any water bottle in the database and use it in their synthesizer.

5. CONCLUSION

We have discussed the synthesis of water bottle acoustics using modal signal processing techniques. We have described the processes of making acoustic measurements of the water bottles, as well as performing modal analysis, with specific considerations for the amount of water contained in the bottle, as well as the stickers placed on the exterior of the bottle. Finally, we have implemented our modal model of a 32 oz. Wide Mouth HydroFlask bottle as a real-time synthesizer plugin.

In this work, we have had to make some assumptions and approximations of the underlying physics. While the results sound compelling, there are many aspects of the water bottle instrument that need more work, such as considering different strike locations and better modelling of the swinging vibrato phenomenon when there is only a very small amount of liquid in the bottle. Future research in this area concerns the extension of water bottle modelling to include a wider range of bottles, with various sizes and shapes, and made of various materials. For high-end water bottle manufacturers, acoustic analysis could be used to improve the sound of their water bottles. Extended research goals include the creation of a physical, musically-tuned water bottle xylophone, as well as potentially determining the optimally acoustic water bottle currently in production.

6. ACKNOWLEDGEMENTS

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