A STRING IN A ROOM: MIXED-DIMENSIONAL TRANSFER FUNCTION MODELS FOR SOUND SYNTHESIS

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ABSTRACT

Physical accuracy of virtual acoustics receives increasing attention due to renewed interest in virtual and augmented reality applications. So far, the modeling of vibrating objects as point sources is a common simplification which neglects effects caused by their spatial extent. In this contribution, we propose a technique for the interconnection of a distributed source to a room model, based on a modal representation of source and room. In particular, we derive a connection matrix that describes the coupling between the modes of the source and the room modes in an analytical form. Therefore, we consider the example of a string that is oscillating in a room. Both, room and string rely on well established physical descriptions that are modeled in terms of transfer functions. The derived connection of string and room defines the coupling between the characteristic string and room modes. The proposed structure is analyzed by numerical evaluations and sound examples on the supplementary website.

1. INTRODUCTION

In virtual acoustics, sound sources are commonly idealized as point sources from which sound radiates away in spherical waves. To add more spatial detail to an omnidirectional source, directivity patterns that govern the direction-dependent radiation characteristics are superposed [1,2]. For a source placed in an enclosed space, the direct path and the excitation of the space depend on the radiation pattern and the source position. In this work, we investigate this phenomenon by not relying on the point source idealization and instead model a spatially distributed vibrating object. As an example, we take an oscillating string placed within a rectangular room (see Fig. 1b) as opposed to a point source radiating the string sound (see Fig. 1a). The string example highlights another aspect of this work: We do not consider arbitrary oscillations, but use the knowledge that strings and rooms behave according to their physical properties constituting specific harmonic modes which themselves are spatially extended shapes [3].

A central contribution of this work is the derivation of a relation between the source and room modes. As such, this work contributes to the physical accuracy of virtual acoustics applications. There are a plethora of vibrating objects which are large relative to human-size, and the missing consideration of near-field effects in their modeling can easily be audible [4]. In particular, virtual and augmented reality (VR/AR) possibly benefits from high-quality realism as complex interactions between sound sources and the surrounding space can often be explored by the audience freely. Similarly, virtual musical acoustic instruments psychoacoustically benefit from modeling their spatial characteristics [5].

In this paper we propose a technique for the interconnection of distributed sources to a room model. The established connection structures are based on a modal representation of the source and the room, i.e., it defines the coupling of the modes of the source and the room modes. As a simple example we consider the oscillation of a one-dimensional (1D) string in a two-dimensional (2D) room. We note that string and room model are based on earlier publications of the authors [6,7]. To the authors' best knowledge, an analytical connection structure that defines the mode coupling of source (string) and room has not been reported, yet. The string and the room are modeled by a transfer function approach that leads to a description in terms of state-space descriptions (SSDs), where the underlying modeling technique is based on modal expansion [8] and is well known in the context of sound synthesis [6,9–11]. The models of the string and room model, are interconnected by adapting techniques from control theory [12,13]. The obtained connection structure reveals to be a matrix that defines the relation between string and room modes which exhibits a strong dependency on the position and the spatial extend of the string in the room. The proposed method is related to the incorporation of complex boundary conditions by feedback loops as proposed by the authors in [6,14] and to the coupling of several instrument parts, e.g., string-bridge coupling [15].

The paper is structured as follows: Section 2 presents the considered scenarios, i.e., a string and a point in a 2D room. The applied modeling technique is defined in Sec. 3 and applied to derive a room model in Sec. 4 and a string model in Sec. 5. Section 6 establishes a connection between the string and the room model for both opposed scenarios. The obtained connection structure is analyzed in Sec. 7 by numerical evaluations. Finally, Sec. 8 concludes the paper and addresses several further works.

2. PROBLEM DESCRIPTION

Before the models of the room and the string and finally their interconnection is established, the geometrical relations of the opposed scenarios of a *string in a room* and a *point in a room* are presented. The exact scenarios that are considered in this paper, are shown in Fig. 1. On the left-hand side Fig. 1a, the oscillation of a string is picked up at a certain position ξ_o and emitted into the room via a point source. Contrarily, the right-hand side Fig. 1b shows the scenario of a string that is oscillating in the room.

In the following, the oscillation of the string is described in terms of its velocity $v_s(\xi, t)$ on a 1D spatial domain with $\xi \in [0, \ell]$

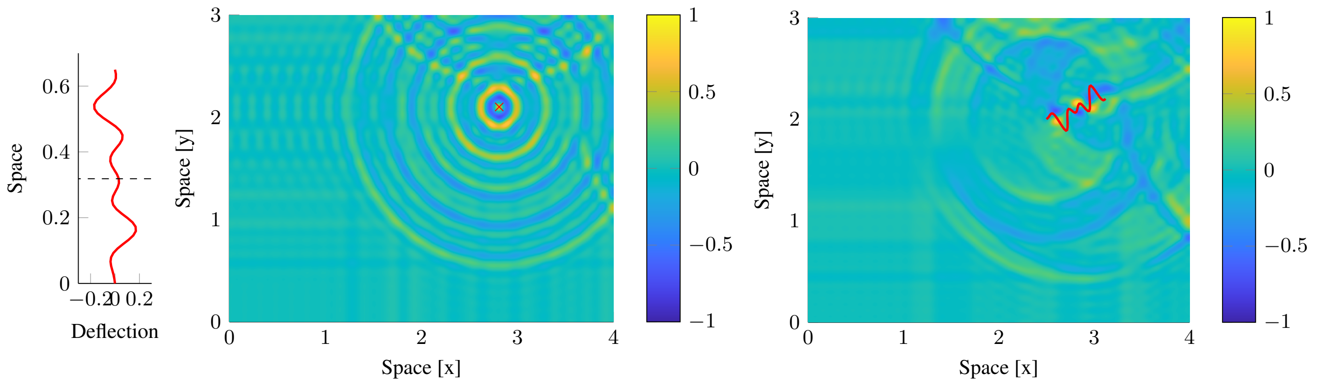


Figure 1: Figure (a) shows the excitation of a room (right) with a string signal $v_\mathrm{s}(\xi_\mathrm{o}, t)$ (left) which is picked from a single point ξ_o (dashed line) and inserted as a point source f_point (red $\times$). Figure (b) shows the excitation of a room with a string signal $v_\mathrm{s}(\xi, t)$ inserted as a distributed line source f_string. Animated versions of both scenarios can be found online on [16].

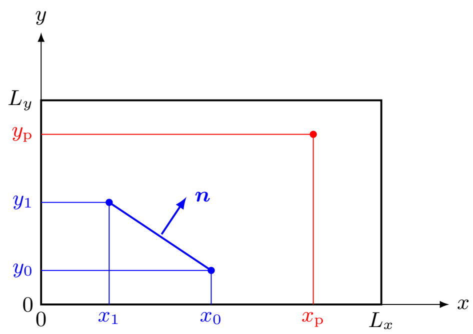


Figure 2: Geometric representation of both scenarios in Fig. 1. Blue: Line source causing a pressure gradient in normal direction $\boldsymbol{n}$ (see Fig. 1b). Red: Point source causing an omnidirectional pressure gradient in the room at a certain position (see Fig. 1a).

and time t. The room is defined on a 2D spatial domain with $\boldsymbol{x} = [x, y]^\mathrm{T}$ and is bounded by fully reflective walls (see Fig. 2).

2.1. Point in a Room

The point source in the room (see Fig. 1a) is spatially defined by a 2D delta impulse at position $\boldsymbol{x}_\mathrm{point} = [x_\mathrm{p},\, y_\mathrm{p}]^\mathrm{T}$

$$\delta(x - x_\mathrm{p})\delta(y - y_\mathrm{p}), \tag{1}$$

which is graphically shown in red in Fig. 2. Together with a temporal behaviour of the source, i.e., the picked up velocity of the string oscillation $v_\mathrm{s}(\xi_\mathrm{o}, t)$, the point excitation function f_point of the room is defined as

$$f_\mathrm{point}(\boldsymbol{x}, t) = \gamma_\mathrm{p}\, v_\mathrm{s}(\xi_\mathrm{o}, t)\, \delta(x - x_\mathrm{p})\delta(y - y_\mathrm{p}). \tag{2}$$

The coupling parameter γ_p is introduced to match the units between the string velocity v_s and the sound pressure in the room. Problem dependent, the parameter γ may be related to physical quantities such as acoustic impedance.

2.2. String in a Room

The oscillation of a string of length ℓ with velocity $v_\mathrm{s}(\xi, t)$ in the room (see Fig. 1b) is defined by a 2D line impulse

$$\delta(\boldsymbol{n}^\mathrm{T}\boldsymbol{x} - c_n) = \delta(n_x x + n_y y - c_n), \tag{3}$$

with normal vector $\boldsymbol{n}$ and

$$c_n = \frac{1}{\ell}\det\left(\begin{bmatrix} x_0 & x_1 \\ y_0 & y_1 \end{bmatrix}\right), \quad \boldsymbol{n} = \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \frac{1}{\ell}\begin{bmatrix} y_1 - y_0 \\ x_0 - x_1 \end{bmatrix}. \tag{4}$$

The geometrical relation between the line impulse and the room is shown in blue in Fig. 2. The 1D nature of the string oscillation is preserved in the 2D room environment by assigning the spatially 1D velocity $v_\mathrm{s}(\xi, t)$ of the string to a spatially 2D function that serves as a weight function for the line impulse in (3)

$$v_\mathrm{s}(x(\xi), y(\xi), t) = \begin{cases} v_\mathrm{s}(\xi, t) & x_1 \le x \le x_0,\ y_0 \le y \le y_1 \\ 0 & \text{else} \end{cases}, \tag{5}$$

where the connection between 2D room coordinates x, y and coordinate ξ along the string is given by coordinate transformation

$$x(\xi) = x_0 + \xi(x_1 - x_0), \qquad y(\xi) = y_0 + \xi(y_1 - y_0). \tag{6}$$

Finally, the excitation of the 2D room by a 1D string of length ℓ is defined by

$$f_\mathrm{string}(\boldsymbol{x}, t) = \gamma_\mathrm{v}\, v_\mathrm{s}(x(\xi), y(\xi), t)\, \delta(\boldsymbol{n}^\mathrm{T}\boldsymbol{x} - c_n). \tag{7}$$

Analogously to (2), a coupling parameter γ_v is introduced.

2.3. Modeling by Transfer Functions

As described in the Introduction, the main focus of this contribution lies on the modeling of the complete interconnected system of the string in the room and to find an expression for the coupling of the individual modes of the string and the room.

The string (see Sec. 5) and the room (see Sec. 4) are separately modeled in terms of transfer functions. Abstractly, the velocity of the string V_s and the sound pressure P of the room are defined

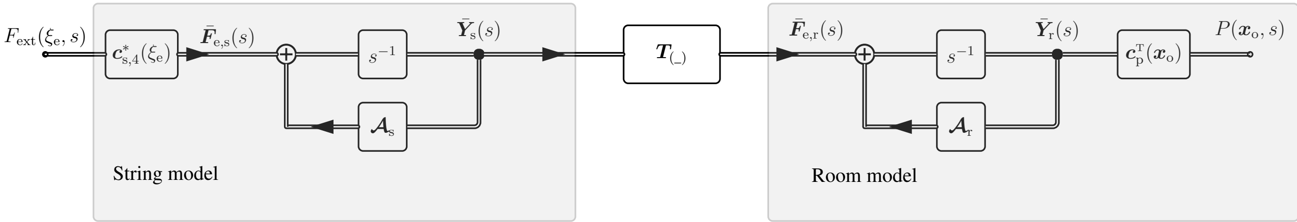


Figure 3: Systematic interconnection of the string and the room by the interconnection of their state space models according to Sec. 6. Systems are connected by the coupling matrix $\boldsymbol{T}_{(\_)}$ that defines the coupling of string and room modes either by a point source $\boldsymbol{T}_{\mathrm{point}}$ or by a string $\boldsymbol{T}_{\mathrm{string}}$.

by the string's and the room's transfer function, respectively, both excited by a suitable excitation function F

$$V_{\mathrm{s}} = H_{\mathrm{string}} \cdot F_{\mathrm{string}}, \qquad P = H_{\mathrm{room}} \cdot F_{\mathrm{room}}. \tag{8}$$

Exploiting the geometrical relations from Sec. 2.2, the goal is to establish a structure T that connects the model of the string with the model of the room (see Sec. 6). Finally, the sound pressure P in the room is expressed by the excitation of the string in the room and their coupled transfer functions

$$P = H_{\mathrm{room}} \cdot T_{\mathrm{string,room}} \cdot H_{\mathrm{string}} \cdot F_{\mathrm{string}}. \tag{9}$$

The interconnection of the room and the string model for both scenarios shown in Fig. 1 is established in Sec. 6 and is graphically shown in Fig. 3.

3. TRANSFER FUNCTIONS FOR nD-SYSTEMS

Both, the room and the string can be mathematically described by initial-boundary value problems (IBVPs). This builds the basis for a modeling procedure in terms of transfer functions, which leads to a system description in terms of SSDs. A detailed description of modeling by transfer functions and SSDs is provided in [6, 10, 14, 17]. In the following the modeling procedure is presented, assuming the most simple boundary and initial conditions.

3.1. Vector-Valued Initial-Boundary Value Problems

A unifying representation of the string and the room is provided in terms of a vector-valued IBVP that is defined on the volume $\boldsymbol{x} \in V$ with boundary $\boldsymbol{x} \in \partial V$ [17]

$$\left[\frac{\partial}{\partial t}\boldsymbol{C} - \mathbf{L}\right] \boldsymbol{y}(\boldsymbol{x}, t) = \boldsymbol{f}_{\mathrm{e}}(\boldsymbol{x}, t), \quad \boldsymbol{x} \in V,\, 0 < t \leq \infty, \tag{10}$$

$$\boldsymbol{F}_{\mathrm{b}}^{\mathrm{H}} \boldsymbol{y}(\boldsymbol{x}, t) = \boldsymbol{\phi}(\boldsymbol{x}, t), \quad \boldsymbol{x} \in \partial V,\, 0 < t \leq \infty, \tag{11}$$

$$\boldsymbol{y}(\boldsymbol{x}, 0) = \boldsymbol{y}_{\mathrm{i}}(\boldsymbol{x}), \quad \boldsymbol{x} \in V,\, t = 0. \tag{12}$$

The $N \times 1$ vector $\boldsymbol{y}$ is the vector of independent variables containing physical quantities describing the underlying system. PDE (10) defines the spatio-temporal dynamics of the system on the volume V, where the spatial derivatives are concentrated in the $N \times N$ spatial differential operator $\mathbf{L}$. The $N \times N$ matrix $\boldsymbol{C}$ in (10) is a capacitance matrix. The $N \times 1$ vector $\boldsymbol{f}_{\mathrm{e}}$ contains excitations of the system. The system behavior at the spatial boundary is defined by a set of boundary conditions (11). A $N \times N$ boundary operator $\boldsymbol{F}_{\mathrm{b}}^{\mathrm{H}}$ is applied to the $N \times 1$ vector $\boldsymbol{y}$ on ∂V yielding a vector of boundary values ϕ. The temporal initial state of the system is defined by a $N \times 1$ vector of initial conditions $\boldsymbol{y}_{\mathrm{i}}$ for the vector $\boldsymbol{y}$ in (12).

The IBVP in (10) – (12) is of general nature and will be specified problem dependent in Secs. 4 and 5. But with a focus on the applications, the boundary conditions are assumed to be homogeneous and the system is at rest for $t = 0$, i.e., $\boldsymbol{\phi} = \mathbf{0}$, $\boldsymbol{y}_{\mathrm{i}} = \mathbf{0}$.

3.2. Laplace Transform

To handle the temporal derivative in (10), a Laplace transform w.r.t. time is applied

$$[s\boldsymbol{C} - \mathbf{L}]\, \boldsymbol{Y}(\boldsymbol{x}, s) = \boldsymbol{F}_{\mathrm{e}}(\boldsymbol{x}, s), \qquad \boldsymbol{x} \in V, \tag{13}$$

$$\boldsymbol{F}_{\mathrm{b}}^{\mathrm{H}} \boldsymbol{Y}(\boldsymbol{x}, s) = \mathbf{0}, \qquad \boldsymbol{x} \in \partial V. \tag{14}$$

Variables in the frequency domain are denoted by uppercase letters and the dependency on the complex frequency variable $s \in \mathbb{C}$.

3.3. Transfer Function Model

The proposed modeling procedure is based on solving the IBVP presented in Sec. 3.2 by modal expansion. Therefore, an infinite set of bi-orthogonal eigenfunctions $\boldsymbol{K}(\boldsymbol{x}, \mu) \in \mathbb{C}^{N\times 1}$ and $\tilde{\boldsymbol{K}}(\boldsymbol{x}, \mu) \in \mathbb{C}^{N\times 1}$ is defined. The functions $\boldsymbol{K}$ are the primal eigenfunctions and $\tilde{\boldsymbol{K}}$ are their adjoints. The discrete spectrum of the spatial differential operator $\mathbf{L}$ is defined by its eigenvalues s_μ [8]. Their exact form is shown in Secs. 4 and 5, but index $\mu \in \mathbb{Z}$ is already introduced now to count the eigenvalues. The modal expansion of PDE (13) leads to a representation of the system in terms of an SSD in a spatio-temporal transform domain [8, 14].

For practical applications, the infinite number of eigenvalues and eigenfunctions has to be restricted. Therefore, the number of eigenvalues is truncated to $\mu = 0, \dots, Q-1$. This leads to a formulation of the transformations in terms of matrices and vectors, instead of infinite sized linear operators [17].

The expansion of PDE (13) is derived by a pair of transformations. A forward transformation $\boldsymbol{\mathcal{T}}$ is defined by a number of Q eigenfunctions $\tilde{\boldsymbol{K}}$, arranged into a $N \times Q$ matrix $\tilde{\boldsymbol{\mathcal{C}}}$ and applied to the vector of variables $\boldsymbol{Y}$ in terms of a scalar product

$$\boldsymbol{\mathcal{T}}\left\{\boldsymbol{Y}(\boldsymbol{x}, s)\right\} = \bar{\boldsymbol{Y}}(s) = \left\langle \boldsymbol{C}\boldsymbol{Y}(\boldsymbol{x}, s), \tilde{\boldsymbol{\mathcal{C}}}(\boldsymbol{x}) \right\rangle, \tag{15}$$

with the vector and the matrix

$$\bar{\boldsymbol{Y}}(s) = \left[\dots, \bar{Y}(\mu, s), \dots\right]^{\mathrm{T}}, \quad \tilde{\boldsymbol{\mathcal{C}}}(\boldsymbol{x}) = \left[\dots, \tilde{\boldsymbol{K}}(\boldsymbol{x}, \mu), \dots\right]. \tag{16}$$

The $Q \times 1$ vector $\bar{\boldsymbol{Y}}$ contains all scalar transform domain representations $\bar{Y}(\mu, s)$ of the vector of variables $\boldsymbol{Y}$

$$\bar{Y}(\mu, s) = \int_V \tilde{\boldsymbol{K}}^{\mathrm{H}}(\boldsymbol{x}, \mu) \boldsymbol{C}\boldsymbol{Y}(\boldsymbol{x}, s)\; \mathrm{d}\boldsymbol{x}. \tag{17}$$

To handle the spatial derivatives in $\mathbf{L}$ in (13), a differentiation theorem is defined [9, 14]

$$\left\langle \boldsymbol{L}\boldsymbol{Y}(\boldsymbol{x},s), \tilde{\boldsymbol{\mathcal{C}}}(\boldsymbol{x})\right\rangle = \boldsymbol{\mathcal{A}}\bar{\boldsymbol{Y}}(s). \tag{18}$$

The $Q \times Q$ matrix $\boldsymbol{\mathcal{A}} = \mathrm{diag}(\ldots, s_\mu, \ldots)$ is the matrix of eigenvalues s_μ of the underlying system.

Finally, the solution $\boldsymbol{Y}$ of PDE (13) is formulated by an inverse transformation $\boldsymbol{\mathcal{T}}^{-1}$ according to (15). It applies a set of Q primal eigenfunctions $\boldsymbol{K}$ arranged in the $N \times Q$ matrix $\boldsymbol{\mathcal{C}}$ to $\bar{\boldsymbol{Y}}$

$$\boldsymbol{\mathcal{T}}^{-1}\{\bar{\boldsymbol{Y}}(s)\} = \boldsymbol{Y}(\boldsymbol{x},s) = \boldsymbol{\mathcal{C}}(\boldsymbol{x})\bar{\boldsymbol{Y}}(s), \tag{19}$$

$$\boldsymbol{\mathcal{C}}(\boldsymbol{x}) = [\ldots, \boldsymbol{K}(\boldsymbol{x},\mu), \ldots] \tag{20}$$

Together, transformation (15) and (19) constitute a forward and inverse Sturm-Liouville transformation [8].

3.4. State-Space Description

Applying the forward transformation (15) to PDE (13) and exploiting the differentiation theorem (18) leads to a representation in the spatio-temporal transform domain

$$s\bar{\boldsymbol{Y}}(s) = \boldsymbol{\mathcal{A}}\bar{\boldsymbol{Y}}(s) + \bar{\boldsymbol{F}}_\mathrm{e}(s). \tag{21}$$

The vector $\bar{\boldsymbol{F}}_\mathrm{e}$ is the transform domain representation of the excitation functions $\boldsymbol{F}_\mathrm{e}$ in (13)

$$\bar{\boldsymbol{F}}_\mathrm{e}(s) = \left\langle \boldsymbol{F}_\mathrm{e}(\boldsymbol{x},s), \tilde{\boldsymbol{\mathcal{C}}}(\boldsymbol{x})\right\rangle. \tag{22}$$

Together, state equation (21) and output equation (19) constitute a state-space description (SSD) that represents the solution of the IBVP and can be used to simulate the dynamics of the physical quantities in $\boldsymbol{Y}$ (see left-hand side of Fig. 3 for the string and right-hand side for the room).

3.5. Interpretation in the Context of Sound Synthesis

This section showed the general procedure of modeling systems – described by an IBVP – in terms of an SSD. Finally, the modeling of systems reduces to the derivation of the matrices $\boldsymbol{\mathcal{C}}$, $\tilde{\boldsymbol{\mathcal{C}}}$ and $\boldsymbol{\mathcal{A}}$ which define the SSD.

The procedure, based on functional transformations and operator theory performs an expansion of the IBVP into a set of eigenfunctions. Particularly, the eigenfunctions $\boldsymbol{K}$ and $\tilde{\boldsymbol{K}}$ in $\boldsymbol{\mathcal{C}}$ and $\tilde{\boldsymbol{\mathcal{C}}}$ describe the spatial behavior of a system and in the context of sound synthesis they represent the spatial shape of the system's modes. In contrast, the system states $\bar{\boldsymbol{Y}}$ describe the temporal behavior of each harmonic, and therefore, state equation (21) defines the temporal evolution of each mode of the system.

4. ROOM MODEL

This section establishes a room model in terms of an SSD according to Sec. 3, which will be connected to the string model in Sec. 6. As already mentioned in Sec. 2 and shown in Figs. 1 and 2, a 2D room with side lengths L_x and L_y is mathematically modeled on the spatial area A

$$A := \{\boldsymbol{x} = [x,y]^\mathrm{T} \,|\, 0 \le x \le L_x,\, 0 \le y \le L_y\}, \tag{23}$$

The dynamics of the sound pressure p and particle velocity $\boldsymbol{v}$ are described by an IBVP, i.e., by the acoustic wave equation on the spatial domain A [7]

$$\rho_0\dot{\boldsymbol{v}}(\boldsymbol{x},t) + \mathrm{grad}\, p(\boldsymbol{x},t) = \boldsymbol{n}\, f_\mathrm{string}(\boldsymbol{x},t), \tag{24}$$

$$\rho_0\, c_0^2 \,\mathrm{div}\, \boldsymbol{v}(\boldsymbol{x},t) + \dot{p}(\boldsymbol{x},t) = f_\mathrm{point}(\boldsymbol{x},t). \tag{25}$$

The parameter ρ_0 denotes the density of air and c_0 is the speed of sound. Temporal derivatives are denoted by a dot, i.e., $\frac{\partial}{\partial t}p = \dot{p}$, while spatial derivatives grad and div denote gradient and divergence in 2D Cartesian coordinates, respectively. The boundary conditions of the room are not shown in detail here, but is assumed, that the walls are fully reflective. Furthermore, it is assumed that the room is at rest for $t = 0$, i.e., $p(\boldsymbol{x}, 0) = 0$.

The variables on the right hand side of (24), (25) are the excitation functions of the room model for both scenarios shown in Fig. 1. The function f_point in (25) denotes an excitation in terms of a sound pressure as a point source in the room (see (2)). The exciation of the room by the string will be modeled by the function f_string (see (7)) that causes a pressure gradient in $\boldsymbol{n}$-direction.

4.1. Unifying Vector Formulation

In order to obtain a transfer function model of the room, the PDEs in (24), (25) are reformulated into a unifying vector formulation according to Sec. 3.1. Exploiting the formulation in (10), the vector of variables and vector of excitation functions end up in (dependencies on time and space are omitted for brevity)

$$\boldsymbol{y}_\mathrm{r} = \begin{bmatrix} p \\ \boldsymbol{v} \end{bmatrix}, \qquad \boldsymbol{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}, \qquad \boldsymbol{f}_\mathrm{e,r} = \begin{bmatrix} \boldsymbol{n}\cdot f_\mathrm{string} \\ f_\mathrm{point} \end{bmatrix}, \tag{26}$$

and the involved matrix and operator read

$$\boldsymbol{C}_\mathrm{r} = \begin{bmatrix} 0 & \rho_0 \\ \frac{1}{\rho_0\, c_0^2} & 0 \end{bmatrix}, \qquad \mathbf{L}_\mathrm{r} = \begin{bmatrix} -\mathrm{grad} & 0 \\ 0 & -\mathrm{div} \end{bmatrix}. \tag{27}$$

The subscript r indicates, that the matrices and vectors are associated with the room model.

4.2. Room: Transfer Function Model

To derive a transfer function model of the room according to Sec. 3.3, not the complete modeling procedure is presented. In Sec. 3 and [6, 17] it has been shown that a system formulated in terms of (10) - (12) can be modeled in terms of a SSD.

Therefore, it is sufficient to derive the necessary contents of the SSD in (19) - (22) which is rewritten here in the continuous frequency domain

$$s\bar{\boldsymbol{Y}}_\mathrm{r}(s) = \boldsymbol{\mathcal{A}}_\mathrm{r}\bar{\boldsymbol{Y}}_\mathrm{r}(s) + \bar{\boldsymbol{F}}_\mathrm{e,r}(s), \tag{28}$$

$$P(\boldsymbol{x},s) = \boldsymbol{c}_\mathrm{p}^\mathrm{T}(\boldsymbol{x})\bar{\boldsymbol{Y}}_\mathrm{r}(s), \tag{29}$$

where the output equation (19) has been reduced to an output equation for the sound pressure p in (29) by reducing the output matrix $\boldsymbol{\mathcal{C}}$ in (20) to its first row.

4.2.1. Eigenvalues

The eigenvalues of the room define the occurring frequencies of the individual room modes and are arranged in the diagonal system matrix $\boldsymbol{\mathcal{A}}_\mathrm{r}$ of the SSD (28). They are derived by evaluating the dedicated dispersion relation of the room and end up in

$$s_\mu = \pm \mathrm{j} c_0 \sqrt{\lambda_x^2 + \lambda_y^2} \qquad \lambda_x = \kappa_x \frac{\pi}{L_x}, \qquad \lambda_y = \kappa_y \frac{\pi}{L_y}, \tag{30}$$

with $\kappa_x, \kappa_y \in \mathbb{Z}$. It has to be mentioned, that the index μ of eigenvalues s_μ directly depends on the wavenumbers κ in x and y-direction, i.e., $(\kappa_x, \kappa_y) \rightarrow \mu$.

4.2.2. Eigenfunctions

According to Sec. 3, there are primal and adjoint eigenfunctions $\boldsymbol{K}_\mathrm{r}$ and $\tilde{\boldsymbol{K}}_\mathrm{r}$. Particularly, the primal eigenfunctions $\boldsymbol{K}_\mathrm{r}$ are arranged in the output matrix $\boldsymbol{\mathcal{C}}_\mathrm{r}$ of the SSD and are used for the transition into the space domain (compare (19)). The adjoint eigenfunctions $\tilde{\boldsymbol{K}}_\mathrm{r}$ are arranged in the matrix $\tilde{\boldsymbol{\mathcal{C}}}_\mathrm{r}$ and are applied for the expansion of a variable into the spatial frequency domain (compare (15)). Both eigenfunctions are derived by the evaluation of their dedicated eigenvalue problems, which is not performed here for brevity. Finally, the eigenfunctions are

$$\boldsymbol{K}_\mathrm{r} = \begin{bmatrix} \cos\lambda_x x \cos\lambda_y y \\ \frac{\lambda_x}{s_\mu \rho_0} \sin\lambda_x x \cos\lambda_y y \\ \frac{\lambda_y}{s_\mu \rho_0} \cos\lambda_x x \sin\lambda_y y \end{bmatrix}, \quad \tilde{\boldsymbol{K}}_\mathrm{r} = \begin{bmatrix} -\frac{\lambda_x}{s_\mu \rho_0} \sin\lambda_x x \cos\lambda_y y \\ -\frac{\lambda_y}{s_\mu \rho_0} \cos\lambda_x x \sin\lambda_y y \\ \cos\lambda_x x \cos\lambda_y y \end{bmatrix}. \tag{31}$$

5. STRING MODEL

This section describes the derivation of a string model in terms of a SSD according to Sec. 3 that will be connected to the room model in Sec. 6. As already mentioned in Sec. 2 and shown in Fig. 1 a 1D string of length ℓ is modeled on the spatial interval L, i.e.,

$$L := \{\xi \big| 0 \leq \xi \leq \ell\}. \tag{32}$$

The dynamics of the string deflection w are described by the well known string equation [18]

$$\rho A \dot{v}_\mathrm{s}(\xi,t) + EI w''''(\xi,t) - T_\mathrm{s} w''(\xi,t) + d_1 v_\mathrm{s}(\xi,t) - d_3 v_\mathrm{s}''(\xi,t) = f_\mathrm{ext}(\xi,t), \tag{33}$$

where v_s is the velocity of the string, i.e., $v_\mathrm{s} = \dot{w}$. Partial derivatives for space ξ are denoted by a prime for brevity, i.e., $\frac{\partial}{\partial \xi} w = w'$. The parameter ρ denotes the string density, A is the cross section area, I is the moment of inertia and T_s is the tension of the string. The constant E denotes Young's modulus. The parameters d_1 and d_3 introduce frequency-independent and frequency-dependent damping into the oscillation of the string. The boundary conditions of the string are not shown in detail, but it is assumed that the string is simply supported at both ends $\xi = 0, \ell$. Furthermore, the string is at rest for $t = 0$, i.e., $w(\xi, 0) = 0$. The function f_ext in (33) represents an excitation function for the string.

5.1. Unifying Vector Formulation

Analogously to Sec. 4.1, the string has to be reformulated into a unifying vector form to derive a string model. Exploiting the formulation in (10), the vector of variables and vector of excitation functions are

$$\boldsymbol{y}_\mathrm{s} = \begin{bmatrix} v_\mathrm{s} & w' & w'' & w''' \end{bmatrix}^\mathrm{T}, \quad \boldsymbol{f}_\mathrm{e,s} = \begin{bmatrix} 0 & 0 & 0 & f_\mathrm{ext} \end{bmatrix}^\mathrm{T}. \tag{34}$$

The matrices $\mathbf{L}_\mathrm{s}, \boldsymbol{C}_\mathrm{s}$ of the vector formulation are not shown in this paper for brevity. A formulation of (10), including all matrices and vectors has been presented by the authors in [6].

5.2. String: Transfer Function Model

In order to obtain a transfer function model of the string it is not necessary to present the complete modeling procedure according to Sec. 3. On the one hand, the procedure has already been shown for a string, e.g., in [6] and on the other hand it is sufficient to derive the necessary contents of the state space description

$$s\bar{\boldsymbol{Y}}_\mathrm{s}(s) = \boldsymbol{\mathcal{A}}_\mathrm{s} \bar{\boldsymbol{Y}}_\mathrm{s}(s) + \bar{\boldsymbol{F}}_\mathrm{e,s}(s), \tag{35}$$

$$V_\mathrm{s}(\xi,s) = \boldsymbol{c}_\mathrm{v}^\mathrm{T}(\xi) \bar{\boldsymbol{Y}}_\mathrm{s}(s), \tag{36}$$

where the output equation (19) has been reduced to an output equation for the string velocity v_s in (36) by reducing the output matrix $\boldsymbol{\mathcal{C}}$ in (20) to its first row.

5.2.1. Eigenvalues

Analogously to the room model, the eigenvalues of the string define the temporal frequencies of the individual modes of the string. They are arranged in the diagonal system matrix $\boldsymbol{\mathcal{A}}_\mathrm{s}$ in (35). The string eigenvalues are derived by the evaluation of the string's dispersion relation and finally end up in [6]

$$s_\nu = -\left(\frac{d_1}{2\rho A} + \frac{d_3}{2\rho A}\gamma_\iota^2\right) \tag{37}$$

$$\pm \mathrm{j} \sqrt{\frac{EI}{\rho A}\gamma_\iota^4 + \frac{T_\mathrm{s}}{\rho A}\gamma_\iota^2 + \left(\frac{d_1}{2\rho A} + \frac{d_3}{2\rho A}\gamma_\iota^2\right)}, \tag{38}$$

with the string's wavenumbers $\gamma_\iota = \iota \frac{\pi}{\ell}$. The index ν of the eigenvalues s_ν depends on the wavenumbers γ_ι. Each γ_ι leads to a complex conjugated pair, i.e., $(\gamma_\iota, \gamma_\iota) \rightarrow (s_\nu, s_{\nu+1})$.

5.2.2. Eigenfunctions

The eigenfunctions of the string determine the spatial shape of each individual mode. Similar to the room model in Sec. 4.2 there are primal and adjoint eigenfunctions. Both are derived by their dedicated eigenvalue problems and end up in [6, 17]

$$\boldsymbol{K}_\mathrm{s} = \begin{bmatrix} \frac{s_\nu}{\gamma_\nu} \sin(\gamma_\nu \xi) \\ \cos(\gamma_\nu \xi) \\ -\gamma_\nu \sin(\gamma_\nu \xi) \\ -\gamma_\nu^2 \cos(\gamma_\nu \xi) \end{bmatrix}, \quad \tilde{\boldsymbol{K}}_\mathrm{s} = \begin{bmatrix} q_1^* \cos(\gamma_\nu x) \\ -\frac{s_\nu^* q_1^*}{\gamma_\nu} \sin(\gamma_\nu \xi) \\ -\gamma_\nu^2 \cos(\gamma_\nu \xi) \\ \gamma_\nu \sin(\gamma_\nu \xi) \end{bmatrix}, \tag{39}$$

with $q_1 = \frac{\rho A s_\nu - d_1}{EI}$.

6. STRING–ROOM COUPLING

In this section, the coupling between the string model (see Sec. 5) and the room model (see Sec. 4) is established by the interconnection of their dedicated SSDs. The goal is to express the excitation function of the room (f_point, f_string) in terms of the system states $\bar{\boldsymbol{Y}}_\mathrm{s}$ of the string model and a suitable connection matrix $\boldsymbol{T}$, which establishes a connection between the individual modes of the string and the modes of the room. The section considers both scenarios defined in Sec. 2, i.e., the room is excited by a string sound replayed by a point source and by a distributed source that is directly placed in the room modeled.

6.1. Point in a Room

The first considered scenario is the excitation of the room by a point source as shown in Fig. 1a. Starting point is the transform domain representation of the excitation function in (28) based on (22), which can be reduced according to the structure of (26)

$$\bar{\boldsymbol{F}}_{\mathrm{e,r}}(s) = \bar{\boldsymbol{F}}_{\mathrm{e,point}}(s) = \left\langle F_{\mathrm{point}}(\boldsymbol{x}, s), \tilde{\boldsymbol{c}}^{\mathrm{T}}_{\mathrm{r},3}(\boldsymbol{x}) \right\rangle, \tag{40}$$

where $\tilde{\boldsymbol{c}}_{\mathrm{r},3}$ is the third line of $\tilde{\boldsymbol{\mathcal{C}}}$ containing the third entries of $\tilde{\boldsymbol{K}}_{\mathrm{r}}$ in (31). The room excitation by a point has been defined in (2) and furthermore, the velocity of the string V_{s} at pickup position ξ_{o} can be rewritten by the output equation (36) of the string model

$$V_{\mathrm{s}}(\xi_{\mathrm{o}}, s) = \boldsymbol{c}^{\mathrm{T}}_{\mathrm{v}}(\xi_{\mathrm{o}})\bar{\boldsymbol{Y}}_{\mathrm{s}}(s). \tag{41}$$

This definition of the velocity and the point impulse position $\boldsymbol{x}_{\mathrm{p}} = [x_{\mathrm{p}}, y_{\mathrm{p}}]^{\mathrm{T}}$ are inserted into (2) and further into (40). Exploiting the sifting property of the delta impulse yields

$$\begin{aligned} \bar{\boldsymbol{F}}_{\mathrm{e,point}}(s) &= \int_A \tilde{\boldsymbol{c}}^{\mathrm{H}}_{\mathrm{r},3}(\boldsymbol{x})\, F_{\mathrm{point}}(\boldsymbol{x}, s)\, \mathrm{d}\boldsymbol{x} \\ &= \gamma_{\mathrm{p}}\, \tilde{\boldsymbol{c}}^{*}_{\mathrm{r},3}(x_{\mathrm{p}}, y_{\mathrm{p}})\, \boldsymbol{c}^{\mathrm{T}}_{\mathrm{v}}(\xi_{\mathrm{o}})\, \bar{\boldsymbol{Y}}_{\mathrm{s}}(s) = \boldsymbol{T}_{\mathrm{point}}\, \bar{\boldsymbol{Y}}_{\mathrm{s}}(s). \end{aligned} \tag{42}$$

The variable $\bar{\boldsymbol{F}}_{\mathrm{e,point}}$ contains the excitations of each individual room mode, which is expressed in terms of the string model's system states $\bar{\boldsymbol{Y}}_{\mathrm{s}}$ and the entries of a coupling matrix $\boldsymbol{T}_{\mathrm{point}}$ which depend on the spatial position of the point source.

6.2. String in a room

The second scenario is the excitation of the room by the string, which is geometrically modeled by a line source as shown in Fig. 1b. Analogously to Sec. 6.1, the starting point is the transform domain representation of the excitation function in (28)

$$\bar{\boldsymbol{F}}_{\mathrm{e,r}}(s) = \bar{\boldsymbol{F}}_{\mathrm{e,string}}(s) = \left\langle \boldsymbol{n}\, F_{\mathrm{string}}(\boldsymbol{x}, s), \begin{bmatrix} \tilde{\boldsymbol{c}}^{\mathrm{T}}_{\mathrm{r},1}(\boldsymbol{x}) \\ \tilde{\boldsymbol{c}}^{\mathrm{T}}_{\mathrm{r},2}(\boldsymbol{x}) \end{bmatrix} \right\rangle, \tag{43}$$

where $\tilde{\boldsymbol{c}}_{\mathrm{r},1}$ and $\tilde{\boldsymbol{c}}_{\mathrm{r},2}$ are the first and the second line of $\tilde{\boldsymbol{\mathcal{C}}}$ containing the first and second entries of $\tilde{\boldsymbol{K}}_{\mathrm{r}}$ in (31). The excitation of the room by a line source has been defined in (7) and the velocity of the string has to be expressed in the room coordinate system according to (5)

$$V_{\mathrm{s}}(x(\xi), y(\xi), s) = \begin{cases} \boldsymbol{c}^{\mathrm{T}}_{\mathrm{v}}(\xi)\bar{\boldsymbol{Y}}_{\mathrm{s}}(s) & x_1 \le x \le x_0,\ y_0 \le y \le y_1 \\ 0 & \text{else} \end{cases}. \tag{44}$$

Inserting this equation into (7), then into the excitation function (43) and exploiting the sifting property of the line impulse and the coordinate transformation in (6) yields

$$\begin{aligned} \bar{\boldsymbol{F}}_{\mathrm{e,string}}(s) &= \int_A \left(n_x\, \tilde{\boldsymbol{c}}^{*}_{\mathrm{r},1}(\boldsymbol{x}) + n_y\, \tilde{\boldsymbol{c}}^{*}_{\mathrm{r},2}(\boldsymbol{x})\right) F_{\mathrm{string}}(\boldsymbol{x}, s)\, \mathrm{d}\boldsymbol{x} \\ &= \gamma_{\mathrm{v}}\, \ell \int_0^1 \left(n_x\, \tilde{\boldsymbol{c}}^{*}_{\mathrm{r},1}(x(\xi), y(\xi)) + \right. \\ &\qquad \left. n_y\, \tilde{\boldsymbol{c}}^{*}_{\mathrm{r},2}(x(\xi), y(\xi))\right) \boldsymbol{c}^{\mathrm{T}}_{\mathrm{v}}(\xi)\, \mathrm{d}\xi\, \bar{\boldsymbol{Y}}_{\mathrm{s}}(s) \\ &= \boldsymbol{T}_{\mathrm{string}} \bar{\boldsymbol{Y}}_{\mathrm{s}}(s). \end{aligned} \tag{45}$$

Analogously to (42), the variable $\bar{\boldsymbol{F}}_{\mathrm{e,string}}$ contains the excitation of each individual room mode and matrix $\boldsymbol{T}_{\mathrm{string}}$ contains the coupling between the spatial modes of the string and the room.

6.3. Transfer Functions

For analysis and comparison of the point and the string excitation of the room, the transfer function between the initial excitation of the string f_{ext} at ξ_{e} and the sound pressure in the room at a pickup position $\boldsymbol{x}_0$ is of particular interest.

According to the definition of the transformed excitation in (22), the term $\bar{\boldsymbol{F}}_{\mathrm{e,s}}$ of the string model in (35) can be derived by exploiting the structure of (34) at an excitation position ξ_{e}

$$\bar{\boldsymbol{F}}_{\mathrm{e,s}}(s) = \left\langle F_{\mathrm{ext}}(\xi_{\mathrm{e}}, s), \tilde{\boldsymbol{c}}^{\mathrm{T}}_{\mathrm{s},4}(\xi_{\mathrm{e}}) \right\rangle = \tilde{\boldsymbol{c}}^{*}_{\mathrm{s},4}(\xi_{\mathrm{e}}) F_{\mathrm{ext}}(\xi_{\mathrm{e}}, s). \tag{46}$$

Then, the transfer function $H_{\mathrm{string}}(\xi_{\mathrm{o}}, s)$ of the string at pickup point ξ_{o} as response to an excitation at point ξ_{e} is derived by inserting (46) into state equation (35) and rearranging

$$H_{\mathrm{string}}(\xi_{\mathrm{o}}, s) = \frac{V_{\mathrm{s}}(\xi_{\mathrm{o}}, s)}{F_{\mathrm{ext}}(\xi_{\mathrm{e}}, s)} = \boldsymbol{c}^{\mathrm{T}}_{\mathrm{v}}(\xi_0)(s\boldsymbol{I} - \boldsymbol{\mathcal{A}}_{\mathrm{s}})^{-1}\tilde{\boldsymbol{c}}^{*}_{\mathrm{s},4}(\xi_{\mathrm{e}}). \tag{47}$$

The excitation of the room by either a point or a string is derived by inserting (46) into state equation (35) of the string. Then, the system states of the string are either inserted into (42) or (45) yielding

$$\bar{\boldsymbol{F}}_{\mathrm{e},(\_)}(s) = \boldsymbol{T}_{(\_)}\, (s\boldsymbol{I} - \boldsymbol{\mathcal{A}}_{\mathrm{s}})^{-1}\, \tilde{\boldsymbol{c}}^{*}_{\mathrm{s},4}(\xi_{\mathrm{e}}) F_{\mathrm{ext}}(\xi_{\mathrm{e}}, s), \tag{48}$$

where $\bar{\boldsymbol{F}}_{\mathrm{e},(\_)}$ and $\boldsymbol{T}_{(\_)}$ are specified either to the point or the string scenario. Inserting (48) into state equation (28) of the room model, subsequently into output equation (29) and rearranging yields the transfer function $H_{\mathrm{room},(\_)}(\boldsymbol{x}_{\mathrm{o}}, s)$ between the pickup of the sound pressure in the room in relation to the excitation of the string F_{ext} either in the point source or in the string scenario

$$\begin{aligned} H_{\mathrm{room},(\_)}(\boldsymbol{x}_0, s) &= \frac{P(\boldsymbol{x}_0, s)}{F_{\mathrm{ext}}(\xi_{\mathrm{e}}, s)} \\ &= \boldsymbol{c}^{\mathrm{T}}_{\mathrm{p}}(\boldsymbol{x}_0)\boldsymbol{H}_{\mathrm{room}}(s)\, \boldsymbol{T}_{(\_)}\, \boldsymbol{H}_{\mathrm{string}}(s)\tilde{\boldsymbol{c}}^{*}_{\mathrm{s},4}(\xi_{\mathrm{e}}), \end{aligned} \tag{49}$$

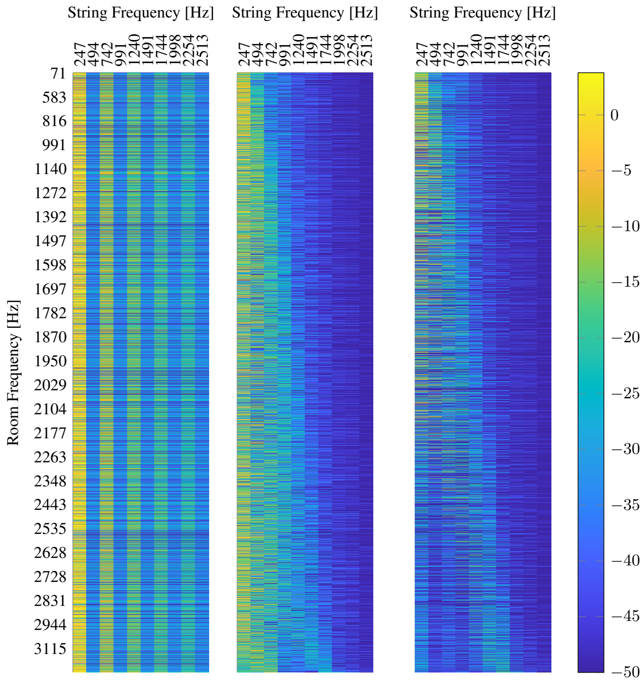
with the matrix-valued mode-wise transfer functions $\boldsymbol{H}_{\mathrm{room}}$ and $\boldsymbol{H}_{\mathrm{string}}$ of the room and the string

$$\boldsymbol{H}_{\mathrm{room}}(s) = (s\boldsymbol{I} - \boldsymbol{\mathcal{A}}_{\mathrm{r}})^{-1}, \quad \boldsymbol{H}_{\mathrm{string}}(s) = (s\boldsymbol{I} - \boldsymbol{\mathcal{A}}_{\mathrm{s}})^{-1}. \tag{50}$$

The established connection of the string and the room model in (49) is graphically shown in Fig. 3 in terms of their SSDs interconnected by the matrix $\boldsymbol{T}_{(\_)}$.

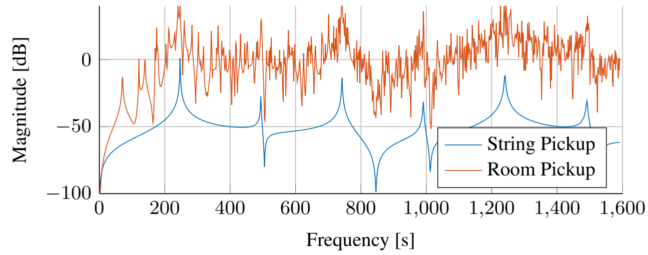
6.4. The Connection Matrix

The connection between the string and the room is established in terms of a connection matrix $\boldsymbol{T}$, which mathematically converts the system states $\bar{\boldsymbol{Y}}_{\mathrm{s}}$ of the string model into excitations for the individual room modes (see Fig. 3). As it can be seen in (42) and in (45) the matrix $\boldsymbol{T}$ itself depends strongly on the spatial eigenfunctions of the room in $\tilde{\boldsymbol{c}}_{\mathrm{r}}$ and of the string in $\boldsymbol{c}_{\mathrm{s}}$ but not on the temporal behavior of the string. Therefore, the matrix describes the spatial coupling between the string and the room in both scenarios described in Fig. 1. Particularly, it defines the possibility of the spatial modes of the source system (point or string) to excite the spatial modes of the room (see Fig. 4).

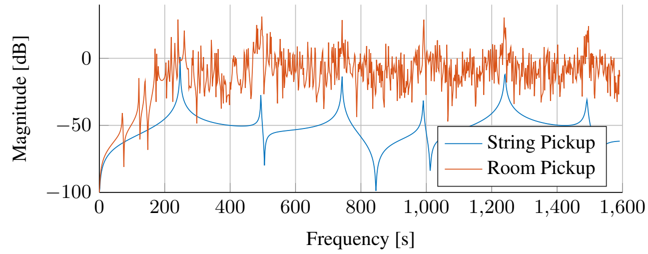


(a) Point in a Room (b) String in a Room (c) Parallel String

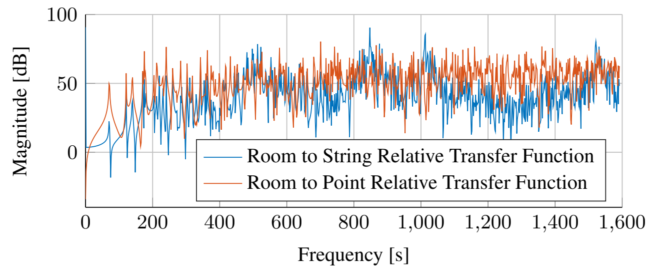
Figure 4: Connection matrices $\boldsymbol{T}_{(\_)}$ as derived in Sec. 6 for the coupling of room modes (y-axis) and the string modes (x-axis). (a): Point source is emitting a string vibration as shown in Fig. 1a. (b): String is placed in the room as shown in Fig. 1b. (c): String is placed parallel to a room wall (e.g., $x_0 = x_1$ in Fig. 2).



(a) Point in a Room



(b) String in a Room



(c) Relative Transfer Function

Figure 5: Transfer functions of the string H_{string} and the room model $H_{\text{room},(\_)}$ for different scenarios (a), (b) and relative transfer functions ${}^{H_{\text{room},(\_)}}/{}_{H_{\text{string}}}$ (c). (a): Point source is emitting a string vibration as shown in Fig. 1a. (b): String is placed in the room as shown in Fig. 1b. (c): Relative transfer functions to see the influence of the connection structure $\boldsymbol{T}_{(\_)}$.

7. NUMERICAL EVALUATION AND ANALYSIS

In this section we present a selection of simulations for the derived model of the string in the room. The results are presented in terms of connection matrices and the room's and string's transfer function. Furthermore, sound examples and videos of the considered scenarios are provided online at [16]. The corresponding MATLAB code to reproduce all figures can be found online[1].

7.1. Scenarios and Parameters

The physics of the room model have been defined in Sec. 4 and the following parameters have been used for evaluation: $L_x = 4\text{m}$, $L_y = 3\text{m}$, $\rho_0 = 1.2\text{kg/m}^3$, $c_0 = 340\text{m}\,\text{s}^{-1}$. Furthermore a number of $Q_{\text{room}} = 2500$ modes is used. As the room model in Sec. 4 does not exhibit any damping, an artificial damping term with adjustable T_{60}-time has been added for the production of videos and sound examples on [16].

The physics of the string have been defined in Sec. 5 and the following parameters have been used for evaluation: $E = 5.4\text{GPa}$, $\rho = 1140\text{kg/m}^3$, $A = 0.5 \cdot 10^{-6}\text{m}^2$, $I = 0.17 \cdot 10^{-12}\text{m}^4$, $d_1 = 8 \cdot 10^{-5}\text{kg}\,\text{m}^{-1}\,\text{s}^{-1}$, $d_3 = 1.4 \cdot 10^{-5}\text{kg}\,\text{m}\,\text{s}^{-1}$, $T_\text{s} = 60.97\text{N}$. The length of the string is $\ell = 0.65\text{m}$, yielding a fundamental frequency of $f_0 = 247\text{Hz}$. Furthermore, a number of $Q_{\text{string}} = 20$ modes is used. The string is excited by a raised-cosine function at $\xi_\text{e} = {}^1/_{\sqrt{2}}$ [6], which models the spatially extended excitation of the string, e.g., by a finger, plectrum or bow.

For the analysis of the established connection structure, two different scenarios are compared as initially shown in Fig. 1:

Point in a room: A point source emitting a string vibration into the room (see Fig. 1a and Sec. 2.1). The point source is placed in the room at the center of the string, i.e., $\boldsymbol{x}_{\text{point}} \approx [2.81\text{m},\ 2.10\text{m}]^\text{T}$. The string is excited at $\xi_\text{e} = {}^1/_{\sqrt{2}}$ and picked up at $\xi_\text{o} = {}^1/_\pi$ to be played into room by the point source.

String in a room: A string placed in the room as a distributed source (see Fig. 1b and Sec. 2.2). The string is placed in the room at $x_1 = 2.5\text{m}$, $x_0 \approx 3.12\text{m}$, $y_0 = 2\text{m}$, $y_1 \approx 2.20\text{m}$.

7.2. Connection matrices

Figure 4 shows the coupling of the room modes and the string modes in terms of the connection matrix $\boldsymbol{T}$ as established in Sec. 6. Figures 4b and 4c show two connection matrices $\boldsymbol{T}_{\text{string}}$ for a string in a room, where in Fig. 4b the string is placed in the room

[1] https://github.com/SebastianJiroSchlecht/MixedDimensionFTM

as shown in Fig. 1b and in Fig. 4c the string is placed parallel to a room wall. Both figures show clearly, that the matrix structure strongly depends on the positioning of the string in the room. Particularly, the positioning of the string influences "how much" a string mode couples to a room mode.

In comparison, Fig. 4a shows the connection matrix $\boldsymbol{T}$ for a string excitation of the room by a point source (see Fig. 1a). It can be seen that nearly all room modes are excited equally by an individual string mode, where the zero lines arise from the positioning of the point source on (or next to) a wave node. This behavior is expected from a point source, as the excitation by an impulse leads to a uniform excitation in the frequency domain.

7.3. Transfer Functions

Figure 5 shows the transfer functions for both considered scenarios, i.e., Fig. 5b shows the transfer function of the room connected to the string by line source as defined in (49) and Fig. 5a shows the same transfer function connected to the string by a point source, both are compared to the string's transfer function in (47). Both figures show, that the room transfer function is shaped by the resonant frequencies of the string. In the *point in a room* scenario in Fig. 5a the entire frequency range of the room transfer function is shaped by string's resonances. In contrast the shaping is not as strong in the *string in a room* scenario in Fig. 5b. Here the shaping is dominant in the lower frequency range and more regularly in the higher frequency range of the room transfer function. Both scenarios confirm the observations in the dedicated connection matrices in Figs. 4a,b. Furthermore, Fig. 5c shows the relative transfer functions between the room transfer functions $H_{\mathrm{room},(\_)}$ and the string H_{string}. This also confirms the previous observations: While the relative transfer function of the *string in a room* exhibits clear peaks, the one of the *point in a room* scenario is nearly flat which confirms the uniform excitation assumption.

8. CONCLUSIONS AND FURTHER WORKS

This paper presented the modeling of a distributed source in a room, i.e., a 1D string oscillating in a 2D room. The model is derived in terms of the interconnection of a string model and a room model by a carefully designed connection structure. The interconnection results in a matrix that describes the coupling of the modes of the source to the room modes in an analytical form depending on the geometric relation of source and room, i.e., the structure defines which mode of a source possibly excites a room mode. Next to the analytical insights from the connection matrix, numerical evaluation showed that the radiation pattern of the distributed source is reflected in the room.

In further works, the concept will be extended to couple also higher dimensional systems, e.g., 2D $\rightarrow$ 3D. In the considered scenario, the string seems to be fully transparent in the room which may be a valid assumption for the 1D $\rightarrow$ 2D coupling. Nevertheless, for higher dimensional coupling, reflections on the surface of the object have to be taken into account as well as the feedback from the room onto the source. The presented room model has been very basic, without any damping and lossless reflections on the wall. Therefore, the extension to more elaborate room models will yield more realistic scenarios.

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